Workshop
on

Optimization problems in intensity-modulated radiation therapy

University of Rostock, 04. – 06. February 2009

Timetable ⧫ Abstracts ⧫ Addresses

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Organizing Committee:

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Dear Participants

of our workshop on

Optimization problems in intensity-modulated radiation therapy

For the treatment of tumors with ionizing radiation, multileaf collimators (MLC) are commonly used for field shaping, which enable an adequate matching between irradiated volume and tumor. A large number of parallel metal leaf pairs can be shifted towards each other from left and right such that parts of the field are covered, while radiation passes through the open region between the leaf pairs. The aim of radiation therapy is to irradiate the tumor with a homogeneous radiation dose while simultaneously sparing surrounding healthy organs and tissue. A variety of practically relevant optimization problems arises in this context.

In a first optimization step, the optimal beam angles and fluence distributions for each angle have to be computed resulting in intensity matrices with nonnegative integer entries. The second step is the decomposition of these matrices into segments, i.e. the computation of a sequence of leaf positions of the MLC, whose superposition amounts to the required intensities. Different objective functions have to be minimized to meet clinical considerations as minimum treatment time and minimum beam-on time. Furthermore, there are some technical constraints that reduce the number of feasible leaf positions. At last, certain segment shapes are inappropriate for dosimetric reasons. A number of combinatorial optimization problems result from these constraints all dealing with an additive decomposition of matrices into 0-1-matrices with specified properties. One can distinguish between problems with exact and approximate decompositions.

In this workshop, the topic and the related optimization problems will be introduced and discussed in detail. The participants report in the talks about their results and their current work. Finally, the most interesting new problems are discussed in small groups, where the main focus is on the clinical relevance of the optimization problems. Participants of the workshop are mathematicians and physicists.

We wish you a fruitful and enjoyable workshop in Rostock and hope that you take pleasant memories home with you.

The organizing committee

Invited talks

Fiorini, Samuel	*Constrained Decompositions of Integer Matrices and their Applications to Intensity Modulated Radiation Therapy* **Change: This talk is presented by Céline Engelbeen.**
Hamacher, Horst W.	*Simultaneous Optimization Methods in IMRT*
Kalinowski, Thomas	*Network flow formulations for approximate shape matrix decomposition*

Talks

Engel, Konrad	*Radiation therapy planning: A challenging optimization problem*
Engelbeen, Céline	*A new exact formulation for the cardinality problem in IMRT*
Gauer, Tobias	*Physical and technical considerations in IMRT*
Kiesel, Antje	*A function approximation approach to the segmentation step in IMRT planning*
Lust, Thibaut	*Biobjective Decomposition of Positive Integer Matrix*

Further participants

Matthias Böhm; Roman Glebov; Cigdem Güler; Johannes Hahn; Marcus Lemke; Liu, Bing; Waldemar Martens; Michael Menrath; Karsten Schölzel; Solvejg Schweder; Meng Xiang-Grüß

Wednesday, 04. February 2009

Room 120

17:00 – 18.00	Registration
18:00 – 19:00	Get together with catering (goulash soup, salad, drinks)
19:00 – 19:30	Welcome and introduction - Antje Kiesel
19.30 – 20.15	Konrad Engel *Radiation therapy planning: A challenging optimization problem*

Thursday, 05. February 2009

Room 120

08.30 – 09.15	Horst W. Hamacher *Simultaneous Optimization Methods in IMRT*
09.15 – 10.00	Samuel Fiorini *Constrained Decompositions of Integer Matrices and their Applications to Intensity Modulated Radiation Therapy* **Change: This talk is presented by Céline Engelbeen.**
10.00 – 10.30	Coffee break
10.30 – 11.15	Thomas Kalinowski *Network flow formulations for approximate shape matrix decomposition*
11.15 – 13.00	Lunch break
13.00 – 13.30	Céline Engelbeen *A new exact formulation for the cardinality problem in IMRT*
13.30 – 14.00	Tobias Gauer *Physical and technical considerations in IMRT*
14.00 – 14.30	Thibaut Lust *Biobjective Decomposition of Positive Integer Matrix*
14.30 – 15.00	Coffee break
15.00 – 15.30	Antje Kiesel *A function approximation approach to the segmentation step in IMRT planning*
16.00 – 17.00	City tour Start at university, afterwards tram to Rostock Südstadt
17.30 – 18:30	Guided tour in the Radiology Department of the "Klinikum für Strahlentherapie" Rostock (Marcus Lemke)
19:00	Dinner

Friday, 06. February 2009

Room 120

08.30 – 11.30 Mathematicians Workshop

Room 31

08.30 – 11.30 Physicists Workshop

Konrad Engel, University of Rostock

Radiation therapy planning: A challenging optimization problem

Intensity-modulated radiation therapy (IMRT) has become an important method in cancer treatment. High energetic radiation delivered by a linear accelerator is used to destroy the tumor, but also affects healthy cells in the surrounding organs. Therefore, the treatment must be planned in such a way that the cancerous cells receive a clinically prescribed dose while the surrounding organs are protected from the radiation as well as possible.

In our talk we give an introduction into the mathematical aspects of the planning process. The task is to find an adequate set of beams, each characterized by a direction, a shape as well as a delivery time of radiation, such that an (almost) optimal treatment can be realized.

We sketch the main aspects of an own software package that includes the whole optimization process, where the dose-, region-, and beam-values are transmitted via the RTOG-format.

Wednesday, 19:30 Room 120

Céline Engelbeen, Université Libre de Bruxelles

A new exact formulation for the cardinality problem in IMRT

We consider a combinatorial problem arising in the segmentation step of the elaboration of a radiotherapy plan. This problem can be described as follows: Given a matrix I with non-negative integer entries, we seek for a decomposition of I as a weighted sum of binary matrices having the consecutive ones property. The objective is first minimize the total sum of the coefficients and, in a second time, the number of matrices used in the decomposition has to be minimized. The coefficients are restricted to be non-negative integers. This problem known in the litterature as the cardinality problem was prouved NP-hard, even if I has only one row (Baatar, Ehrgott, Hamacher, Johnston, 2005) or one column (Collins, Kempe, Saia, 2007). We investigate an exact formulation of the problem. For this, we build a network where the number of nodes depends on the values of the coefficients of I. So, we first use it for matrices where the values of all entries are between 1 and 5. This formulation is under research now. Moreover, with the help of this network we can prove that the cardinality problem is polynomial when the biggest value in I is bounded by B for a matrix I of any size.
Keywords: Decomposition of integer matrices, consecutive ones property, multileaf collimator, radiation therapy.

Samuel Fiorini, Université Libre de Bruxelles

Constrained Decompositions of Integer Matrices and their Applications to Intensity Modulated Radiation Therapy

We consider combinatorial optimization problems arising in radiation therapy. Given a matrix I with non-negative integer entries, we seek a decomposition of I as a weighted sum of binary matrices having the consecutive ones property, such that the total sum of the coefficients is minimized. The coefficients are restricted to be non-negative integers. Here, we investigate variants of the problem with additional constraints on the matrices used in the decomposition. Constraints appearing in the application include the interleaf motion and interleaf distance constraints. The former constraint was previously studied by Baatar *et al.* (*Discrete Appl. Math., 2005*) and Kalinowski (*Discrete Appl. Math., 2005*). The latter constraint was independently considered by Kumar *et al.* (*working paper, 2007*) in the case where coefficients of the decomposition are not restricted to be integers. For both constraints, we prove that finding an optimal decomposition reduces to finding a maximum value potential in an auxiliary network with integer arc lengths and no negative length cycle. This allows us to simplify and unify the previous approaches. Moreover, we give an $O(MN+KM)$ algorithm to solve the problem under the interleaf distance constraint, where M and N respectively denote the number of rows and columns of the matrix I and K is the number of matrices used in the decomposition. We also give an $O(MN \log M + KM)$ algorithm for solving the problem under the interleaf motion constraint and hence improve on previous results. Finally, we show the problem can still be solved in $O(MN \log M + KM)$ time when both constraints are considered simultaneously.

Tobias Gauer, Department of Radiotherapy and Radiation Physics, University Medical Center Hamburg-Eppendorf

Physical and technical considerations in IMRT

The basic idea of IMRT is effortless to understand. The intensity of incoming beams is adapted to the shape of the tumour which results (theoretically) in high conformity between irradiated volume and tumor volume. To verify the resulting treatment plans, however, effort must be made as most of the physical and technical impacts are ignored in current IMRT planning systems. The optimization is done as if all elements of the planning and delivery chain work without a loss in plan quality. As a matter of fact, there are a few uncertainties and negative impacts on IMRT planning. In addition to the problem of physiological organ motions, uncertainty exists in the dose calculation model. Furthermore, dose delivery is affected by field shaping with multileaf collimators. This talk turns towards physical and technical considerations in IMRT and presents an IMRT technique that enables the integration of these constraints in order to make the resulting plan robust against uncertainties.

Horst W. Hamacher, Technische Universität Kaiserslautern

Simultaneous Optimization Methods in IMRT

Mathematical optimization has contributed in the last 10 years considerably to the development of high quality radiation plans concentrating mainly on three classes of problems,

- the geometry problem,
- the intensity problem, and
- the realization problem.

In general, these three problems are solved sequentially. In the geometry problem, a best selection of angles is identified from which radiation is delivered. The output of the geometry problem is used (and never changed) to determine an intensity profile for each of the angles. In the last problem, the chosen intensity profiles are realized using, for instance, multileaf collimators. The talk consists in two parts: We will first give a survey of methods which are used to tackle the three problems. This material is based on a paper of the authors together with Mathias Ehrgott and Lizhen Shao, University of Auckland, available online in the journal 4OR. Particular emphasis is given in this part to possibilities of extending current methods to the simultaneous treatment of all or at least two of these problems. The second part will be devoted to the latter issue by describing in more detail a combinatorial optimization approach dealing with the simultaneous intensity and realization problem.

Thomas Kalinowski, University of Rostock

Network flow formulations for approximate shape matrix decomposition

One step in the IMRT planning process is the decomposition of the desired fluence distribution into a set of deliverable homogeneous fields. The discretized fluence distribution is represented by a nonnegative integer matrix, and the deliverable fields correspond to certain binary matrices, called shape matrices. An important objective is the minimization of the total irradiation time. In the talk we consider a possible tradeoff between the total irradiation time and the exactness of the decomposition. In other words, we ask how much the irradiation time can be reduced when we allow some deviations from the prescribed fluence values. In a second step we ask how much the fluence has to be changed in order to be deliverable in a given irradiation time. We reformulate this problem as a min cost flow problem on a network with $O(mn)$ nodes and $O(mn)$ arcs where $m \times n$ is the size of the given fluence matrix.

Antje Kiesel, University of Rostock

A function approximation approach to the segmentation step in IMRT planning

In the segmentation step of IMRT planning, given intensity profiles have to be decomposed into a number of leaf positions of a multileaf collimator (MLC) such that the superposition of the corresponding field shapes is close to the desired profile. Until now, these decomposition problems have been formulated as discrete optimization problems where the profiles are nonnegative integer matrices. The segments were modeled as 0-1-matrices, 1 indicating that radiation is transmitted through this part of the field and 0 for the areas that are covered by the leaves of the MLC. But in physical reality, radiation has a penumbra at the boundary of the segment causing a decline of the intensity, that is not modeled in these formulations. This paper embeds the segmentation task into the wider context of function approximation and models both profiles and segments as real-valued functions of two variables. This leads to convex optimization problems whose objective is to minimize the approximation error between the profile and the superposition of the segments with their real weights. Thus, a more realistic model of radiation is used and may enable an improvement in treatment quality.

Thibaut Lust, Faculté Polytechnique de Mons

Biobjective Decomposition of Positive Integer Matrix

We consider the following problem: to decompose a positive integer matrix into a linear combination of binary matrices that respect the consecutive ones property. The positive integer matrix corresponds to fields giving the different radiation beams that a linear accelerator has to send throughout the body of a patient. Due to the inhomogeneous dose levels, leaves from a multi-leaf collimator are used between the accelerator and the body of the patient to block the radiation. The leaves positions can be represented by segments, that are binary matrices with the consecutive ones property. The aim is to find a decomposition that minimizes the irradiation time and the setup-time to configure the multi-leaf collimator at each step of the decomposition. We propose for this NP-hard biobjective problem a heuristic method, based on the Pareto local search method. Experimentations are lead on different size instances and the results are reported. These first results are encouraging and are a good basis for the design of more elaborated methods.