ORTHOGONAL DOUBLE COVERS OF GENERAL GRAPHS

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Abstract. Let H be a graph on n vertices and $\mathcal{G}$ a collection of n subgraphs of H, one for each vertex. $\mathcal{G}$ is an orthogonal double cover (ODC) of H if every edge of H occurs in exactly two members of $\mathcal{G}$ and any two members share an edge whenever the corresponding vertices are adjacent in H.
ODCs of complete graphs have been widely studied in literature. In this paper, we are concerned with ODCs of arbitrary graphs. In particular, we investigate the existence of ODCs whose members are isomorphic sets of independent edges.

1. Introduction

Let K_n be the complete graph on n vertices and $V(K_n)$ its vertex set. An *orthogonal double cover* (ODC) of the complete graph K_n is a collection $\mathcal{G}$ of subgraphs G_i, $i \in V(K_n)$, such that every edge of K_n occurs in exactly two subgraphs and any two distinct subgraphs have precisely one edge in common. Throughout, the subgraphs in $\mathcal{G}$ are said to be the *pages* of the ODC. A simple counting argument shows every page to contain exactly $n-1$ edges. If all pages are isomorphic copies of a given graph G, we speak of an ODC by G.

Orthogonal double covers have received much attention during the last two decades. Their investigation was originally motivated by problems from statistical design theory [20] and the theory of Armstrong databases [7]. The general question under discussion is the following: Given a graph G, decide whether there is an ODC of K_n by G. Although far from being solved completely, this question gave rise to a considerable number of results on ODCs by special graphs such as paths [18, 19, 21, 25, 22], trees [13, 23, 24], cycles [6, 11, 12], graphs with maximum degree two [3, 5, 15] or clique graphs [2, 10]. Moreover, the notion of an ODC has been generalized in various directions, like orthogonal decompositions of complete digraphs [12, 17], suborthogonal double covers [16, 30], or symmetric graph designs [4, 9, 14].

In this paper, we are going to study orthogonal double covers of an arbitrary underlying graph, which gives us yet another generalization of the original notion of an ODC of K_n. Let H be a given graph with vertex set $V(H)$. An *orthogonal double cover* (ODC) of H is a collection $\mathcal{G}$ of subgraphs G_i, $i \in V(H)$, of the graph H such that every edge of H occurs in exactly two subgraphs, and any two subgraphs G_i, G_j share an edge iff the vertices i, j are adjacent in H. Again, the members of $\mathcal{G}$ are said to be *pages*. If all pages are isomorphic copies of a graph G, then $\mathcal{G}$ is called an ODC of H by G.

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Clearly, the existence of isolated vertices in H or G has no impact on the existence of an ODC of H by G, unless the vertex number of G exceeds the vertex number of H. Therefore, we shall consider all graphs in this paper to be without isolated vertices. Moreover, we denote the vertex number and the edge number of a graph F by $\nu(F)$ and $\epsilon(F)$, respectively.

Every ODC of a graph H induces a bijection π on the edge set $E(H)$ of H: for every edge $e = \{i, j\}$, the pages $\mathcal{G}_i$ and $\mathcal{G}_j$ share a (uniquely determined) edge e', and we put $\pi(e) = e'$. On the other hand, every bijection π on $E(H)$ yields an ODC of H: its page G_i is the subgraph consisting of the edges $\pi(e)$, where e runs through all edges incident with vertex i in H. If, in particular, π is the identity bijection, page G_i is just the star with centre i in H. Clearly, the edge number of this star equals the degree of i in H. This gives rise to the following simple observation.

Proposition 1. *There is an ODC of a given graph H by some graph G iff H is regular.*

At this point, it is natural to ask, whether there exists a similar characterization of all graphs that are able to play the role of G in an orthogonal double cover by G.

Theorem 2. *For every graph G there is some graph H admitting an ODC by G.*

Proof. Let Z_2^r be the elementary abelian 2-group of order 2^r. Its elements are the r-tuples with entries from $\{0, 1\}$. By o we denote the all-zero tuple, and by u_c the tuple with a one in component c, and zeros everywhere else.

For every pair $\{i, j\}$ of elements from Z_2^r we define its length by $\ell(\{i, j\}) = i + j$. For a graph F with vertices from Z_2^r, its length set $L(F)$ is the set of all elements $z \in Z_2^r$ such that there is an edge of length z in F.

Let H be a graph with vertex set $V(H) = Z_2^r$, and G with vertex set $V(G) \subseteq Z_2^r$. We define the page G_i, $i \in Z_2^r$, as the isomorphic copy of G under the mapping $x \to x + i$ for every $x \in Z_2^r$. It not difficult to see, that the collection $\mathcal{G}$ of all these pages forms an ODC of H by G iff any two distinct edges of G have different length, and H contains a pair $\{i, j\}$ of elements from Z_2^r as an edge precisely when its length $\ell(\{i, j\})$ lies in the length set $L(G)$ of G. An ODC obtained in this way is said to be *generated* by Z_2^r.

Now, let G be an arbitrary graph, and put $r = \nu(G) - 1$. Without loss of generality, we may assume, that G has vertex set $V(G) = \{o, u_1, \ldots, u_r\}$. Then, any two distinct pairs of vertices from G are of different length. Finally, let H be the graph with vertex set $V(H) = Z_2^r$ consisting of all edges with length in $L(G)$. Consequently, there is an ODC of H by G which is generated by Z_2^r. □

For any two graphs F and F', let $F \cup F'$ denote the vertex disjoint union of these graphs. Moreover, let rF be the vertex-disjoint union of r copies of F, and $K_n \backslash F$ be the complement of F. Clearly, the existence of ODCs of two graphs H and H' by G yields the existence of an ODC of $H \cup H'$ by G. Hence, for every G there are infinitely many graphs admitting an ODC by G.

2. ODCs by forests

As pointed out earlier, in an ODC of the complete graph K_n, every page contains $n-1$ edges. This gives a special motivation for investigating ODCs of K_n by trees. However, to construct an ODC by a given tree turned out to be difficult already for paths, although there are results in this direction. On the other hand, Gronau, Mullin and Rosa [13] conjectured the existence of an ODC of the complete graph K_n by every tree on n vertices with one genuine exception: there is no ODC of K_4 by the path P_4. So far, this conjecture has been verified for all $n \leq 14$ [24].

To begin with, we are going to prove an analogue of the Gronau-Mullin-Rosa conjecture for ODCs of hypercubes.

Theorem 3. *For every forest G with $\epsilon(G) = r$ edges, there exists an ODC of the r-dimensional hypercube Q_r by G, with one genuine exception, namely $2K_2$.*

Proof. Assume, Q_r has vertex set $V(Q_r) = Z_2^r$, and its edges are exactly those pairs with length in $\{u_1, \dots, u_r\}$. Our objective is to construct an ODC by G, which is generated by Z_2^r. Thus, we have to find a copy of G as a subgraph of Q_r, containing exactly one edge of length u_c for every $c = 1, \dots, r$.

For $r \leq 3$, this problem can easily be settled whenever G is different from $2K_2$. Thus, we assume $r \geq 4$ and proceed by induction on r.

Given a graph F, a vertex is *pendent* if it is of degree 1, and an edge is *pendent* if it is incident with a pendent vertex. In addition, if both vertices are pendent, we call the edge *independent*.

Let e be a pendent edge in the forest G. On deleting e and all its pendent vertices, we obtain a subgraph G' with $r-1$ edges. By the induction hypothesis, there is copy of G' as a subgraph of Q_{r-1} with the claimed condition on the lengths. Consequently, there is a copy of G' as a subgraph of Q_r, whose vertices all have a zero in component r, and whose edges are of lengths $u_1, \dots, u_{r-1}$.

It remains to add an edge of length u_r such that the resultant subgraph is a copy of G. If the missing edge e is not independent in G, then this can be done in a straightforward way. Conversely, if e is independent in G, we can choose its copy of length u_r whenever we have $\nu(G') < 2^{r-1}$. But this holds, since $\nu(G') \leq 2\epsilon(G') = 2(r-1) < 2^{r-1}$ is true for every $r \geq 4$.

This concludes the induction and the proof of the claim. $\square$

It is well-known, that there is no ODC of the complete graph K_4 by the path P_4 on 4 vertices (cf. [19]). However, there is a large variety of other graphs H that admit an ODC by P_4. Of course, each of these graphs has to be 3-regular. A k-factor in a graph F is a k-regular spanning subgraph of F. The question, whether a 3-regular graph admits a 1-factor, and thus a 2-factor too, goes back to Petersen [26]. As shown by Robinson and Wormald [27], almost every 3-regular graph has an edge decomposition into a 1-factor and a 2-factor. For these graphs we obtain the following result.

Theorem 4. *There is an ODC by the path P_4 for every 3-regular graph containing a 1-factor and without a component isomorphic to K_4.*

Proof. Let H be a 3-regular graph without copies of K_4 as components, and admitting an edge decomposition into a 1-factor H' and a 2-factor H''.

To begin with, consider the cycle C_n on n vertices. Assume its vertex set to be the cyclic group Z_n, such that its edges are the pairs $\{i, i+1\}$, $i = 0, \ldots, n-1$. Let π be the bijection defined by $\pi(\{i-1, i\}) = \{i, i+1\}$, see Figure 1. In the corresponding ODC, page G_i consists of the edges $\{i, i+1\}$ and $\{i+1, i+2\}$. This gives us an

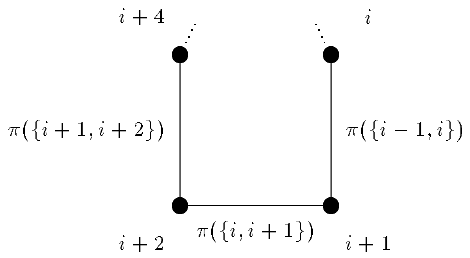

FIGURE 1. The bijection π corresponding to an ODC of C_n by P_3.

ODC of C_n by the path P_3. As every 2-regular graph is the vertex-disjoint union of cycles, we obtain an ODC of the 2-factor H'' by P_3 such that every vertex i is a pendent vertex on the page G''_i.

On extending every subgraph G''_i by the unique edge containing vertex i in the 1-factor H', we obtain an ODC of H whose pages are copies of P_4 or C_3. Pages isomorphic to C_3 are said to be *bad.* Let π be the corresponding bijection on the edge set $E(H)$. Our objective is to replace π by a slightly modified version, which gives us the claimed ODC by P_4.

Suppose, there is a vertex i such that its page G_i is bad. We denote the neighbors of i by a, b and c, such that $\{a, i\}$ and $\{i, b\}$ are edges in the 2-factor, $\{i, c\}$ is an edge in the 1-factor, and the page G_i is the cycle with vertices i, b and c. Moreover, let d denote the second neighbor of c in the 2-factor. We shall distinguish five cases.

(*First case.*) Suppose i has more than one neighbor whose corresponding page is bad. It is not difficult to see, that the union of these bad pages forms a copy of K_4. This, however, has been excluded by assumption. Henceforth, we may assume that i has at most one neighbor admitting a bad page.

(*Second case.*) If neighbor a admits a bad page, then we restart the discussion with vertex a playing the role of i. As pointed out in the first case, i will be the only neighbor of a admitting a bad page. Thus, without loss of generality, we may assume, that a does not admit a bad page.

(*Third case.*) Suppose, none of the neighbors of i admits a bad page. Then, we redefine π by exchanging $\pi(\{i, c\})$ and $\pi(\{b, c\})$, see Figure 2. Consequently, we replace edge $\{i, c\}$ by $\{c, d\}$ in page G_i, and vice versa in page G_b. As a result, G_i

is the path with vertices i, b, c, d, while G_b stays a path, too. All other pages remain unmodified.

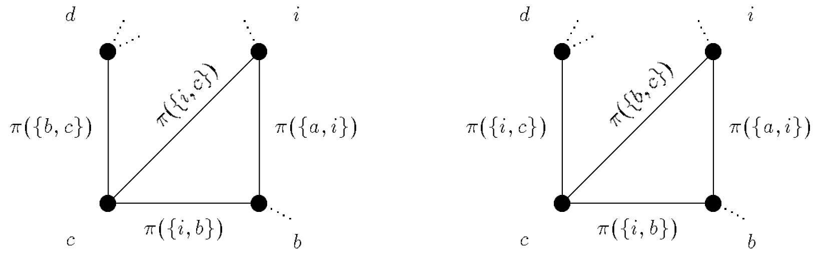

FIGURE 2. Suggested modification of π in the third case.

(*Fourth case.*) Suppose b is the only neighbor of i admitting a bad page. Then, G_b is the cycle with vertices b, c and d, where $\{b, d\}$ is an edge from the 1-factor. Again, we redefine π by exchanging $\pi(\{i, c\})$ and $\pi(\{b, c\})$, see Figure 3. As a result, G_i is the path with vertices i, b, c, d, and G_b is the path with vertices i, c, b, d. All other pages remain unchanged.

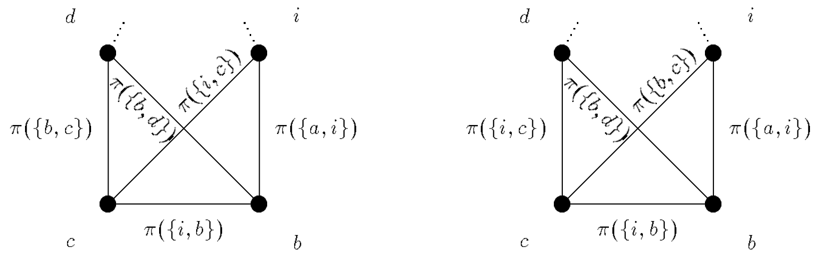

FIGURE 3. Suggested modification of π in the fourth case.

(*Fifth case.*) Suppose, c is the only neighbor of i with a bad page. Since G_c contains the edges $\{i, c\}$ and $\{c, d\}$, the edge $\{d, i\}$ must appear as an edge in the 2-factor. Thus, G_c is the cycle with vertices c, d and i. This time, we redefine π by exchanging $\pi(\{i, b\})$ and $\pi(\{b, c\})$, see Figure 4. Hence, we replace edge $\{b, c\}$ by $\{c, d\}$ in page G_i, and vice versa in page G_c. As a result, G_i is the path with vertices b, i, c, d, while G_c is the path with vertices b, c, i, d. Again, all other pages remain as before.

In each of the cases, we modify only two pages: G_i and either G_b or G_c. Afterwards, each of these three pages is a path. This reduces the number of bad pages by at least 1.

It is noteworthy, that the vertices i, b and c are exactly those on the former bad page G_i. After the modification, none of them lies on a bad page any longer. Hence, we may apply the same modification to the next bad page in the ODC. In the end, we obtain an ODC without bad pages, i.e. an ODC by P_4 as claimed. $\square$

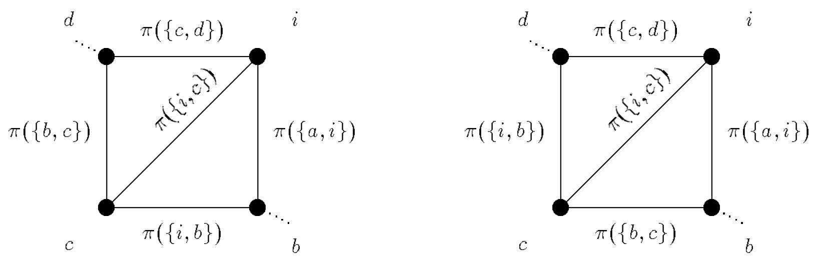

FIGURE 4. Suggested modification of π in the fifth case.

In the remainder of this section, we restrict ourselves to the forests rK_2, where r is some positive integer. Our objective is to prove the existence of an ODC by rK_2 for every sufficiently large r-regular graph H. We make use of the following result on packings of graphs into complete graphs, due to Sauer and Spencer [29]. Two graphs F and F', with n vertices each, are said to be *mutually placeable* if there are edge-disjoint copies of F, F' as subgraphs of the complete graph K_n.

Theorem 5 (Sauer and Spencer). *Let F and F' be two graphs, both with vertex number n, and maximum degree d and d', respectively. They are mutually placeable whenever $2dd' < n$ holds.*

Theorem 6. *Every r-regular graph H with vertex number $\nu(H) \geq \frac{16(r-1)^2+2}{r}$ admits an ODC by rK_2.*

Proof. Assume, there is an ODC $\mathcal{G}$ of an r-regular graph H, and let π be the corresponding bijection on the edge set of H. Then, $\mathcal{G}$ is an ODC by rK_2 iff the edges of every page are mutually disjoint, i.e. iff $\pi(e)$ and $\pi(e')$ are disjoint for every pair of distinct edges e, e' with non-empty intersection in H. Hence, to find the claimed ODC it suffices to give a suitable bijection on $E(H)$.

Let L be the line graph of H. Its vertex set is $V(L) = E(H)$, and there is an edge connecting e and e' in L whenever they are adjacent in H. A bijection π on $E(H)$ is suitable for the claimed ODC iff L and its isomorphic copy under the bijection π are edge-disjoint. Thus, the existence of a suitable bijection π merely depends on the question whether two copies of the line graph L are mutually placeable or not.

Since H is r-regular, each of its edges is incident with exactly $2(r-1)$ other edges. This shows the line graph L to be $2(r-1)$-regular. Applying the Theorem of Sauer and Spencer, we conclude that two copies of L are mutually placeable whenever

$$8(r-1)^2 + 1 \leq \nu(L) = \epsilon(H) = \tfrac{r}{2}\nu(H)$$

holds, which implies the claimed result. □

Note, that the bound in Theorem 6 is usually not sharp. Trivially, every graph admitting an ODC by rK_2 has at least $2r$ vertices. A well-known example of an r-regular graph on $2r$ vertices is the complete bipartite graph $K_{r,r}$ with two partition classes of size r each. In an ODC of $K_{r,r}$, the pages indexed by the vertices of only one partition class form an edge decomposition of $K_{r,r}$.

A k-factorization of a graph F is an edge decomposition into k-factors. Thus the existence of an ODC of $K_{r,r}$ by rK_2 corresponds to the existence of two 1-factorizations of $K_{r,r}$ such that any two 1-factors from different factorizations share exactly one edge. As pointed out by Alspach, Heinrich and Liu [1], this again corresponds to the existence of a pair of orthogonal Latin squares of order r, which gives rise to the following observation. For further results on ODCs of complete bipartite graphs, we refer to the forthcoming paper [8].

Proposition 7. *There exists an ODC of the complete bipartite graph $K_{r,r}$ by rK_2 iff r is different from 2 and 6.*

For 2-regular graphs, i.e. families of vertex-disjoint cycles, we obtain the following corollary to Theorem 6. Throughout, let C_n denote the cycle on n vertices.

Corollary 8. *There exists an ODC by $2K_2$ for every 2-regular graph H with three genuine exceptions: C_3, C_4 and $2C_3$.*

Proof. It suffices to check the claim for graphs with at most 8 vertices. Let Z_n denote the cyclic group of order n, and assume C_n to have vertex set $V(C_n) = Z_n$ such that its edges are just the pairs $\{i, i+1\}$, $i = 0, \dots, n-1$. Moreover, let $z \in Z_n$ be coprime to n and different from 1, $n-1$. Then, the bijection π defined by $\pi(\{i, i+1\}) = \{zi, zi+1\}$ is suitable for the existence of an ODC of C_n by $2K_2$. It is easy to check, that such an element z exists iff we have $\phi(n) \geq 2$, where ϕ denotes the Euler totient function. This is known to hold for $n = 5$ and every $n \geq 7$. The

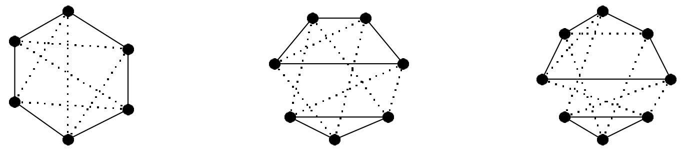

FIGURE 5. Solutions for C_6, $C_3 \cup C_4$ and $C_3 \cup C_5$.

non-existence of an ODC for the three mentioned exceptions is obvious. Hence, we have to study three remaining graphs, namely C_6, $C_3 \cup C_4$ and $C_3 \cup C_5$. For each of them, Figure 5 shows two mutually placeable copies of the line graph. □

Robinson and Wormald [28] proved that almost all regular graphs with even vertex number have a 1-factor. To conclude this section, we present the following observation for these graphs, which combines ideas from Theorems 4 and 6.

Lemma 9. *Let H be an r-regular graph with an (r-1)-factor H'', such that H'' admits an ODC by a given graph G''. Then there is also an ODC of H by $G'' \cup K_2$, whenever $\nu(H) \geq 4\nu(G'') + 8\epsilon(G'')$ holds.*

Proof. Let π'' be the bijection on the edge set $E(H'')$ corresponding to the given ODC of H''. In addition, let π' be any bijection on the edge set of the 1-factor. On composing both, we obtain a bijection on the edge set $E(H)$ which gives us an ODC of H.

Every page G_i of the resultant ODC arises from the page G_i'' of the original ODC by adding a new edge from the 1-factor. We call a page G_i *bad* if the additional edge is not independent in G_i. Our objective is to show that there is a bijection π' that does not produce any bad page. This, of course, would prove the claim of the lemma.

Thus, among all possible bijections π', we take one that causes the minimum number of bad pages. Let G_i be one of these bad pages, and let $e = \{i, j\}$ be the edge containing i from the 1-factor. We aim on redefining π' such that neither G_i nor G_j stay bad pages, and such that the modification does not produce new bad pages. Actually, we shall do this by exchanging $\pi'(e)$ with $\pi'(f)$, where f is a suitable edge from the 1-factor.

On the one hand, we have to choose f such that $\pi'(f)$ is not incident with any vertex of G_i or G_j. This condition is satisfied by all edges from the 1-factor with at most $2\nu(G'')$ exceptions. Clearly, e itself is among them.

On the other hand, $\pi'(e)$ should not be incident with any vertex of a page G_x'', where x is on the edge f. Suppose, that every vertex of H'' lies on at most p pages. Then, the second condition holds for all edges from the 1-factor with at most $2p$ exceptions. Again, e is one of them.

Clearly, the existence of a suitable edge is guaranteed if the 1-factor contains sufficiently many edges, i.e. if

$$\tfrac{1}{2}\nu(H) \geq 2\nu(G'') + 2p. \tag{1}$$

This, however, holds by assumption since p is bounded from above by $2(r-1)$, and $\epsilon(G'')$ equals $r-1$. □

As an application of Lemma 9, we obtain the following observation.

Theorem 10. *There is an ODC by $P_3 \cup K_2$ for every 3-regular graph H with a 1-factor and $\nu(H) \geq 24$.*

Proof. Lemma 9 immediately yields the existence for graphs H with at least 28 vertices. However, we can do it slightly better: In an ODC of the 2-factor H'' by P_3 every vertex occurs on exactly $p = 3$ pages. Equation (1) provides the claimed result for graphs with at least 24 vertices. □

3. ODCs by small complete graphs

Theorem 11. *A graph H admits an ODC by K_3 iff each of its components is isomorphic to K_4.*

Proof. Assume, there is an ODC of H by K_3. Every vertex in H lies on exactly three pages, which mutually share an edge. Hence, their union yields a subgraph isomorphic to K_4 in H. But, H is 3-regular itself, and therefore every component of H must be a copy of K_4. On the other hand, the graph rK_4 admits an ODC by K_3 for every positive integer r: let the page G_i be the unique copy of K_3 in the same component of rK_4 like i, but not containing i as a vertex. $\square$

Theorem 12. *A graph H admits an ODC of K_4 iff each of its components is isomorphic to K_7 or $K_8 \backslash 4K_2$.*

Proof. Assume, there is an ODC of H by K_4. Then H is 6-regular, and every vertex i occurs in precisely 4 pages. Any two of them share at most one edge, and it is easy to check, that this forces them to have mutually exactly one edge in common. The union of these pages forms a subgraph isomorphic to $K_7 \backslash 4K_2$ in H. We distinguish two cases: either the component of H containing vertex i has only 7 vertices, or there is some vertex j which is adjacent with a neighbor of i, but not with i itself. Evidently, the component under discussion is isomorphic to K_7 in the first case. In the second case, however, it turns out that the additional vertex j must be adjacent with every neighbor of i, due to the definition of an ODC. This gives us a component isomorphic to $K_8 \backslash 4K_2$.

On the other hand, the existence of an ODC of the complete graph K_7 by K_4 is well-known (cf. [2]). It remains to construct an ODC of the graph $K_8 \backslash 4K_2$ by K_4. Let the vertex set of $K_8 \backslash 4K_2$ be the elementary 2-group Z_2^3, such that the missing edges are those of length $u_1 = (1,0,0)$. Moreover, assume K_4 to be the complete graph with vertices $o = (0,0,0)$, $u_2 = (0,1,0)$, $u_3 = (0,0,1)$ and $u_1 + u_2 + u_3 = (1,1,1)$. Then, there is an ODC of $K_8 \backslash 4K_2$ by K_4 generated by Z_2^3. $\square$

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