Pairwise Balanced Designs whose Block Size Set Contains Seven and Thirteen

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Abstract

In this paper, we investigate the PBD-closure of sets K with $\{7, 13\} \subseteq K \subseteq \{7, 13, 19, 25, 31, 37, 43\}$. In particular, we show that $v \equiv 1 \bmod 6$, $v \geq 98689$ implies $v \in B(\{7, 13\})$. As a preliminary result, many new 13-GDDs of type 13^q and resolvable BIBD with block size 6 or 12 are also constructed. Furthermore, we show some elements to be not essential in a Wilson bases for the PBD-closed set $\{v : v \equiv 1 \bmod 6, v \geq 7\}$.

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1 Introduction

Let K be a set of positive integers. Then a *pairwise balanced design* $\mathrm{PBD}(v, K)$ of order v with block sizes from K is a pair $(V, \mathcal{B})$, where V is a finite set (the *point set*) of cardinality v and $\mathcal{B}$ is a family of subsets (called *blocks*) of V which satisfy the following properties:

(i) every pair of distinct elements of V occurs in exactly one block of $\mathcal{B}$;

(ii) if $B \in \mathcal{B}$, then $|B| \in K$.

In a sequence of three papers R.M. Wilson [25, 26, 28] developed a theory of PBD-closed sets and the notation of PBD-closure. A set S of positive integers is said to be *PBD-closed* if the existence of a $\mathrm{PBD}(v, S)$ $(v > 0)$ implies that v belongs to S. Let K be a set of positive integers and let $B(K) = \{v > 0 : \exists\ \mathrm{PBD}(v, K)\}$. Then $B(K)$ is a PBD-closed set called the *PBD-closure* of K. According to Wilson's theory there exists a constant $c_0(K)$ such that designs $\mathrm{PBD}(v, K)$ exist for all $v \geq c_0(K)$ which satisfy the congruences $(v-1) \equiv 0 \bmod \alpha(K)$ and $v(v-1) \equiv 0 \bmod \beta(K)$, where $\alpha(K) = \gcd\{k-1 : k \in K\}$ and $\beta(K) = \gcd\{k(k-1) : k \in K\}$. Concerning the structure of PBD-closed sets Wilson also showed that if S is a PBD-closed set, then S is *eventually periodic* with period $\beta(S)$; that is, there exists a constant $c_0(S)$ such that for every $k \in S$, $\{v : v \geq c_0(S), v \equiv k \bmod \beta(S)\} \subseteq S$.

The theory of PBD-closed sets is a powerful tool for investigations of combinatorial structures, a finite number of known examples of a certain set of objects can establish the existence of the entire set of these objects, for examples, see [22, p. 203] and [27].

Unfortunately, the constant $c_0(K)$ is not known in general. Although Wilson's proof is somehow constructive, the estimate of the constant is very large. Therefore, one attempts to determine $B(K)$ for given K as accurately as possible, for a survey, see, for example, [22, Tables 3.17, 3.18].

In this paper, we investigate the PBD-closure of sets K with $\{7, 13\} \subseteq K \subseteq \{7, 13, 19, 25, 31, 37, 43\}$. Our motivation to consider these sets K comes from a problem on pan-orientable block designs. In [19] it is shown that the set of values v which admit a pan-orientable block design $(v, 4, 2)$ is a PBD-closed set containing 7 and 13 and is a subset of $N_{1,6} = \{v : v \geq 6, v \equiv 1 \bmod 6\}$. We are able to give in Section 5 an upper bound on $c_0(\{7, 13\})$: $c_0(\{7, 13\}) \leq 98689$. More precise upper bounds are obtained when looking at the residues modulo 42. In Table 1 we present the largest possible exception in each residue class $6t + 1$ modulo 42, $0 \leq t < 7$. Note that the largest exceptions in the residue classes $1, 7$ or 13 modulo 42 are small

$6t+1$ modulo 42	1	7	13	19	25	31	37
Largest exception	2605	1645	14293	82549	98683	91507	88447

Table 1: Largest possible exception in each residue class $6t+1$ modulo 42

compared to largest exceptions for $19, 25, 31$ or 37 modulo 42. Therefore, it seems opportune to include in K the smallest values from each fibre modulo 42 in order to eliminate numbers that are in the list of exceptions for $B(\{7, 13\})$. So, we study in Section 6 the PBD-closure of sets K where $\{7, 13\} \subset K \subseteq \{7, 13, 19, 25, 31, 37, 43\}$.

The constructions we used to establish these results are partially taken from Mullin and Stinson [23] which determined the PBD-closure of $P_{1,6}$, where $P_{1,6}$ is defined to be the set of prime powers congruent 1 modulo 6. They showed that $B(P_{1,6}) \cup Q = N_{1,6}$, where Q is a set of 31 (possible) exceptions. Subsequently, their result was improved by Greig [17] who removed 9 of the values in Q. In Section 4 a number of new constructions, which are based on Wilsons Fundamental Construction, are introduced. As ingredients for these constructions we will construct many new 13-GDDs of type 13^q and (resolvable) BIBD with block size $6, 12$ or 13.

As a consequence of Wilson's existence theory it follows that if S is a PBD-closed set, then there exists a finite subset $J \subseteq S$ such that $S = B(J)$. Such a set J is said to be a *finite basis* or *Wilson basis* for the closed set S. An element $x \in J$ is called *essential* if and only if $x \notin B(J \setminus \{x\})$. Mullin [21] determined a first Wilson basis for the set $N_{1,6} = B(\{$7, 13, 19, 25, 31, 37, 43, 55, 61, 67, 73, 79, 97, 103, 109, 115, 121, 127, 139, 145, 157, 163, 181, 193, 199, 205, 211, 223, 229, 235, 241, 253, 265, 271, 277, 283, 289, 295, 307, 313, 319, 331, 349, 355, 361, 367, 373, 379, 391, 397, 409, 415, 421, 439, 445, 451, 457, 487, 493, 499, 643, 649, 655, 661, 667, 685, 691, 697, 709, 727, 733, 739, 745, 751, 781, 787, 811, 1063, 1069, 1231, 1237, 1243, 1249, 1255, 1315, 1321, 1327, 1543, 1549, 1567, 1579 1585, 1783, 1789, 1795, 1801, 1819, 1831$\})$. Independently, Du [14] and Greig [17] removed the elements 223, 253, 295, 307, 361, 367, 379, 421, 439, 655, 727, 1231, 1237, 1243, 1249, 1255, 1543, 1549, 1585, 1783, 1789, 1795, 1801, 1819 and 1831 from the finite basis for $N_{1,6}$. We will improve this result in Section 4 by showing that 4 further values 1063, 1069, 1567 and 1579 are not essential.

2 Preliminaries

In this section, we give some definition and notations as well as some preliminary results which will be used in the sequel. We refer the reader to [9] and [12] for undefined terms as well as a general overview of design theory.

Fundamental to our constructions are a number of designs which we define now. A *group-divisible design* (GDD) is a triple $(V, \mathcal{G}, \mathcal{B})$ where V is a set of *points*, $\mathcal{G}$ is a partition of V into *groups* and $\mathcal{B}$ is a collection of subsets of V (called *blocks*) such that any pair of distinct points in V occurs together either in some group or in exactly one block, but not both. A K-GDD of type $g_1^{t_1} g_2^{t_2} \ldots g_r^{t_r}$ is a GDD in which each block has size from the set K and in which there are t_i groups of size $g_i, i = 1, 2, \ldots, r$. We will denote a $\{k\}$-GDD as a k-GDD. Group divisible designs are useful as ingredients in Wilson's Fundamental Construction.

Construction 2.1 *Let $(V, \mathcal{G}, \mathcal{B})$ be a GDD and let $w : V \to \mathbb{Z}^+ \cup \{0\}$ (w is called a* weight *function). Suppose that for each block $B \in \mathcal{B}$ there is a K-GDD of type $\{w(x) : x \in B\}$. Then there is a K-GDD of type $\{\sum_{x \in G_i} w(x) : G_i \in \mathcal{G}\}$.*

A *transversal design* $\mathrm{TD}(k, n)$ is a k-GDD of type n^k. It is well-known that the existence of a TD (k, n) is equivalent to that of $k-2$ mutually orthogonal Latin squares (MOLS) of side n. Also, it is well known that for all prime powers q there is a $\mathrm{TD}(q+1, q)$. Our source of TDs is the following result of MacNeish [20].

Lemma 2.2 *If a TD(k, m) and a TD(k, n) exist, then a TD(k, mn) exists.*

Moreover, we use the full knowledge of the updated MOLS table [4, 13, 6] which provides a list of lower bounds on the number of MOLS of all orders up to 10000. In particular, $\mathrm{TD}(14, m)$ play an important role in one of our constructions. Thus, we mention the following result [4, Table 2.68].

Lemma 2.3 *If $m \geq 6713$ and $m \equiv 0$ mod 7, or if $m \geq 3567$ and m is odd, or if $m \geq 7289$, then there is a TD$(14, m)$.*

We also require the notion of incomplete designs. An *incomplete pairwise balanced design* $\mathrm{IPBD}(v, h, K)$ is a triple $(V, Y, \mathcal{B})$ where V is a set of v points, Y is a subset of V of size h (Y is called the *hole*), and $\mathcal{B}$ is a collection of subsets of V (blocks) such that

(i) any pair of distinct elements of V occur together either in the hole Y or in exactly one block of $\mathcal{B}$, but not both;

(ii) if $B \in \mathcal{B}$, then $|B| \in K$.

Suppose $(V, Y, \mathcal{B})$ is an IPBD(v, h, K) and $(Y, \mathcal{Y})$ is a PBD(h, K). Then we say that $(V, \mathcal{B} \cup \mathcal{Y})$ is a PBD with a *flat* Y of order h. Note that in any PBD any block can be considered to be a flat.

Lemma 2.4 *Suppose there is an IPBD*(v, w, K) *and an IPBD*(w, h, K)*. Then there is an IPBD*(v, h, K)*.*

Proof. Fill the hole W of the IPBD(v, w, K) with the blocks of the IPBD(w, h, K) defined on the point set W to obtain the desired design. □

An *incomplete transversal design* ITD$(k; n, m)$ is a quadruple $(V, Y, \mathcal{G}, \mathcal{B})$ where V is a set of kn points, Y is a subset of V of size km (Y is called the *hole*), $\mathcal{G}$ is a partition of V into k groups, each of size n, and $\mathcal{B}$ is a collection of k-subsets of V (blocks) such that

(i) for each group $G_i \in \mathcal{G}$, $|G_i \cap Y| = m$,

(ii) for each group $G_i \in \mathcal{G}$ and each block $B_j \in \mathcal{B}$, $|G_i \cap B_j| = 1$, and

(iii) any pair of points from distinct groups occur together either in the hole Y or in exactly one block, but not both.

Here, we list five criteria for determining the existence of ITDs which follow from results of Brouwer and van Rees [10].

Lemma 2.5 *Suppose there exists a TD*(k, m)*, a TD*$(k, m+1)$*, a TD*$(k+1, t)$*, and* $0 \leq u \leq t$*. Then there exists an ITD*$(k; mt + u, u)$*.*

Lemma 2.6 *Suppose there exists a TD*(k, m)*, a TD*$(k, m+1)$*, a TD*$(k, m+2)$*, a TD*$(k+2, t)$*, a TD*(k, u) *and* $0 \leq u, v \leq t$*. Then there exists an ITD*$(k; mt + u + v, v)$*.*

Lemma 2.7 *Suppose there exists a TD*(k, m)*, a TD*$(k, m+1)$*, a TD*$(k, m+2)$*, a TD*$(k+u+1, t)$*, a TD*$(k+1, m+u)$ *and* $0 \leq v \leq t-1$*. Then there exists an ITD*$(k; mt + u + v, v)$*.*

Lemma 2.8 *Let* $m > 1$ *and suppose there exist a TD*(k, m)*, a TD*$(k, m+1)$ *and a TD*$(k+u, t)$*. Then there exists an ITD*$(k; mt + u, m + u)$*.*

Lemma 2.9 *If there exists a TD*(k, m)*, then there exists an ITD*$(k; m, 1)$*.*

3 Known Designs

In this section, we collect together previously constructed designs for later use.

A PBD$(v, \{k\})$ is usually called a *balanced incomplete block design* BIBD$(v, k, 1)$. *Resolvable* BIBDs are designs that admit a partition of the block set into subsets of blocks that contain every point exactly once. These designs are denoted by RBIBD$(v, k, 1)$. RBIBDs will be used to construct IPBDs and PBDs. Adjoining a new point to each partition of an RBIBD$(v, k, 1)$ yields an IPBD$(v + (v-1)/(k-1), (v-1)/(k-1), \{k+1\})$ and, if $\{k+1, (v-1)/(k-1)\} \subset B(K)$, a PBD$(v + (v-1)/(k-1), K)$.

BIBD$(v, 7, 1)$ have been extensively studied for a long time. The following result is the culmination of the contributions of several authors, for a survey see, for example, [3].

Lemma 3.1 *A BIBD$(v, 7, 1)$ exists if $v \equiv 1$ or 7 mod 42 except when $v = 43$ and possibly when $v = 42t + 1$ and $t \in \{2, 3, 5, 6, 12, 14, 17, 19, 22, 27, 33, 37, 39, 42, 47, 59, 62\}$ or $v = 42t + 7$ and $t \in \{3, 19, 34, 39\}$.*

Resolvable BIBDs can be constructed from block disjoint difference families by the Ray-Chaudhuri-Wilson construction, see [24]. Difference families with block size 6 (which can all be made block disjoint) have been investigated by Chen and Zhu [11].

Lemma 3.2 ([11, 24, 18]) *Let $q = 30t + 1, q \neq 61$ be a prime power. Then there exists a BIBD$(q, 6, 1)$ obtained from a block disjoint difference family, an RBIBD$(6q, 6, 1)$ and an IPBD$(216t + 7, 36t + 1, \{7\})$.*

We also know the following finite set of RBIBD$(v, 6, 1)$s.

Lemma 3.3 ([2, 16, 18]) *If $4 \leq t \leq 832$, t is even and $6t + 1$ is a prime power, or if $6 \leq t \leq 296$, t is even and $5t + 1 \not\equiv 1$ mod 50 is a prime power, or if $t \in \{5, 57\}$, then an RBIBD$(30t + 6, 6, 1)$ exists and hence an IPBD$(36t + 7, 6t + 1, \{7\})$.*

Furthermore, we mention the following group divisible designs.

Lemma 3.4 ([8, 7, 3]) *There exist a 7-GDD of type 3^{15} and 7-GDDs of type 7^n for $n \equiv 1$ mod 6, $n \notin \{19, 115, 145, 205, 235\}$.*

Lemma 3.5 *If $v \in B(\{7\})$, then there is a 7-GDD $6^{\frac{v-1}{6}}$.*

Proof. Delete just one arbitrary point in a BIBD$(v, 7, 1)$. □

Lemma 3.6 *There exist a* $\{7,13\}$*-GDD* 6^{51} *and a* 7*-GDD* $6^{49}12^1$*.*

Proof. There is a PBD$(307,\{7,13\})$ with exactly one block of size 13, see for example [17, Lemma 13.11] or Lemma 4.19 ($u=7, v=49, w=7, a=6$). Delete a point not on the 13-block for the GDD of type 6^{51} or a point from the 13-block for the GDD of type $6^{49}12^1$. $\square$

Lemma 3.7 ([18]) *Let* t *be an integer with* $t \notin \{2-6,\ 10-14,\ 16,\ 18-27,\ 29-32,\ 34,\ 35,\ 37-40,\ 42-48,\ 51-55,\ 59-62,\ 93-95,\ 98,\ 100-103,\ 107-111,\ 116-118,\ 138,\ 139,\ 146,\ 152,\ 154,\ 156-160,\ 163-167,\ 170-174,\ 177-181,\ 185-189,\ 191,\ 192,\ 194,\ 195,\ 199,\ 200,\ 201,\ 207-209,\ 215,\ 216,\ 219,\ 221,\ 228-230,\ 269,\ 270,\ 272,\ 275-278,\ 283,\ 285,\ 286,\ 326,\ 334,\ 339,\ 342\}$*. Then there is a* 7*-GDD of type* 42^t*. If* $\{7,43\} \subset K$*, then there is a PBD*$(42t+1,K)$*.*

Some useful group divisible designs arise from transversal designs. A proof of the following lemma is given in [23, Lemmas 2.1, 2.2].

Lemma 3.8 *If there is a TD*(k,n)*, then there is a* $\{k,n\}$*-GDD of type* $(k-1)^n(n-1)^1$*. If there is a TD*$(k,n-1)$*, then there is a* $\{k,n\}$*-GDD of type* $(k-1)^{n-1}(n-1)^1$*.*

Corollary 3.9 *There exist a* $\{7,13\}$*-GDD* $6^{13}12^1$ *and a* $\{7,13\}$*-GDD* $6^{12}12^1$*. There exist a* 7*-GDD* $6^{49}48^1$ *and a* 7*-GDD* $6^{48}48^1$*.*

Proof. The first two GDDs are obtained from a TD$(7,13)$ and a TD$(7,12)$, respectively. The next two GDDs are obtained from a TD$(7,49)$ and a TD$(7,48)$, respectively. Note that blocks of size 49 are replaced by blocks of a BIBD$(49,7,1)$. $\square$

The following is a result of Greig [17, Thm. 8.4] which is based on a construction of Brouwer using Baer subplanes.

Lemma 3.10 *If* q *is a prime power, and* $0 < t < q^2-q+1$*, then it follows that* $t(q^2+q+1)+q^2-q+1-t \in B(\{t+1, q+t, (q^2-q+1-t)^*\})$ *and that there exists an IPBD*$(t(q^2+q+1)+q^2-q+1-t,\ q^2-q+1-t,\ \{t+1,q+t\})$*.*

4 Constructions

From GDDs, we construct IPBDs by filling in the groups by appropriate ingredient IPBDs.

Lemma 4.1 *Suppose there is a K-GDD of type $g_1, g_2, \ldots, g_n$. If we have an IPBD($g_i + f, f, K$) for $2 \leq i \leq n$, and we have a PBD($g_1 + f, K$), then there is a PBD($G + f, K$) (where $G = \sum_{i=1}^{i=n} g_i$) containing a flat of order $g_1 + f$.*

In the case our GDD is a TD, we get the following specialization of Lemma 4.1.

Lemma 4.2 *Suppose there is a TD(k, n). If $\{k, n\} \subset B(K)$, then there is a PBD(kn, K) containing flats of order k and n. If $\{k, n+1\} \subset B(K)$, then there is a PBD($kn + 1, K$) containing flats of order k and $n + 1$.*

The second construction is a simple application of Wilson's Fundamental Construction.

Lemma 4.3 *Suppose there is a TD(t, m) and a k-GDD of type a^t. If $\{k, am\} \subset B(K)$, then there is a PBD(atm, K) containing flats of order k and am. If $\{k, am+1\} \subset B(K)$, then there is a PBD($atm+1, K$) containing flats of order k and $am + 1$.*

Proof. Take the TD(t, m), a t-GDD m^t as the master design in Wilson's Fundamental Construction, apply weight a to all points and replace the blocks of size t by the k-GDD of type a^t. In the first case use a PBD(k, K) and a PBD(am, K) to replace the blocks of size k and to fill the groups of size am. In the second case adjoin a common point to all groups and fill these groups with a PBD($am + 1, K$). □

The next constructions (Lemmas 4.4, 4.5, 4.6 and 4.10) are taken from [23]. They are applications of Wilson's Fundamental Construction together with filling in the groups by appropriate designs.

Lemma 4.4 *Suppose $\{k, n, m(k-1)+1, m(n-1)+1, t(k-1)+1\} \subset B(K)$, a TD($k, n$), a TD($k, n-1$), and a TD($n+1, m$) exists, and $0 \leq t \leq m$. Then there is a PBD($(n-1)mk + t(k-1) + 1, K$) containing flats of order $k, n, m(k-1)+1, m(n-1)+1$ and $t(k-1)+1$.*

Corollary 4.5 *Suppose there is a TD($14, m$) and $0 \leq t \leq m$. If $\{7, 13, 6m+1, 12m+1, 6t+1\} \subset B(K)$, then there is a PBD($84m+6t+1, K$) containing flats of order $7, 13, 6m+1, 12m+1$ and $6t+1$.*

Lemma 4.6 *Suppose there exist K-GDDs of group types 1^{n+1} and $1^n c^1$. If there is a TD($n+1, m$) and $0 \leq t \leq m$, then there exists a K-GDD of group type $m^n(t(c-1)+m)^1$.*

Corollary 4.7 *Suppose $\{u, m, 6t+m\} \subset B(K)$ and a TD(u, m) exists. If there is a PBD($u + 6, K$) with a flat of order 7, and $0 \leq t \leq m$, then there exists a PBD($mu + 6t, K$) with flats of order m and $6t + m$.*

Proof. Take $n = u - 1$ and $c = 7$ in Lemma 4.6, noting that a PBD(u, K) can be considered as a K-GDD 1^u and a PBD$(u + 6, K)$ containing a flat of order 7 can be considered as a K-GDD $1^{u-1}7^1$. Then fill in groups. □

Corollary 4.8 *Suppose $u, m, 12t + m \in B(K)$ and a TD(u, m) exists. If there is a PBD$(u+12, K)$ with a flat of order 13, and $0 \leq t \leq m$, then there exists a PBD$(mu + 12t, K)$ with flats of order m and $12t + m$.*

Proof. Take $n = u - 1$ and $c = 13$ in Lemma 4.6. □

Lemma 4.9 *Suppose $\{u, m, 6t+m\} \subset B(K)$ and a TD(u, m) exists. If there is a PBD$(u + 6, K)$ with a flat of order 7, a PBD$(u + 12, K)$ with a flat of order 13, and $0 \leq t \leq 2m$, then there exists a PBD$(mu + 6t, K)$ with flats of order m and $6t + m$.*

Proof. Write $t = x + 2y$ with $x + y \leq m$. Give all points in the TD(u, m) weight 1, except for x and y points in the last group, which get weight 7 and 13, respectively. Apply Wilson's Fundamental Construction using K-GDD 1^u, K-GDD $1^{u-1}7^1$ and K-GDD $1^{u-1}13^1$ derived from given PBDs. Then fill in groups. □

Lemma 4.10 *Suppose $\{n, m(n - 1) + 1\} \subset B(K)$, a TD$(n, n)$, and a TD$(n, m)$ exists. Then there is a PBD$((n^2 - 1)m + 1, K)$ containing flats of order n and $m(n - 1) + 1$.*

In the following we prove a few further applications of Wilson's Fundamental Construction which are useful for our purposes.

Lemma 4.11 *Let $\{7, 13\} \subset B(K)$, suppose there is a TD$(14, m)$ and $0 \leq t \leq m$. If there exist an IPBD$(6m + f, f, K)$, and*

a) an IPBD$(12m + f, f, K)$ and $6t + f \in B(K)$, or

b) an IPBD$(6t + f, f, K)$ and $12m + f \in B(K)$, or

c) an IPBD$(12m + f, f, K)$, an IPBD$(6t + f, f, K)$ and $6m + f \in B(K)$,

then there is a PBD$(84m + 6t + f, K)$ containing flats of order $7, 13$ and a flat of order a) $6t + f$, or b) $12m + f$, or c) $6m + f$.

Proof. Truncate all but t points of the last group of the TD$(14, m)$ to obtain a $\{13, 14\}$-GDD $m^{13}t^1$. Giving weight 12 to all points of one untruncated group and weight 6 to all other points Wilson's Fundamental Construction yields a $\{7, 13\}$-GDD $(6m)^{12}(12m)^1(6t)^1$ where we use as ingredient GDDs a $\{7, 13\}$-GDD $6^{12}12^1$ and a $\{7, 13\}$-GDD $6^{13}12^1$ from Corollary 3.9. Now applying Lemma 4.1 completes the proof. □

Lemma 4.12 *Let* $\{7, 13\} \subset B(K)$ *and suppose there is a TD*$(14, m)$*. If there exist an IPBD*$(6(m-1)+f, f, K)$

a) and $12m + f \in B(K)$*, or*

b) an IPBD$(12m + f, f, K)$ *and* $6(m-1) + f \in B(K)$*,*

then there is a PBD$(90m - 78 + f, K)$ *containing flats of order* $7, 13$ *and a flat of order a)* $12m + f$*, or b)* $6(m-1) + f$*.*

Proof. Truncate all but one point of a block of the TD$(14, m)$ to obtain a $\{13, 14\}$-GDD $m^1(m-1)^{13}$. Giving weight 12 to all points of the group of size m and weight 6 to all other points Wilson's Fundamental Construction yields a $\{7, 13\}$-GDD $(12m)^1(6(m-1))^{13}$ where we use as ingredient GDDs a $\{7, 13\}$-GDD $6^{12}12^1$ and a $\{7, 13\}$-GDD $6^{13}12^1$ from Corollary 3.9. Again, applying Lemma 4.1 completes the proof. □

Lemma 4.13 *Let* $\{7, 13\} \subset B(K)$*, suppose there is a TD*$(t+1, m)$*, assume* $\{6t+1, 6t+7\} \subset B(\{7\})$ *and* $0 \le r \le m$*. If there exist an IPBD*$(6m+f, f, K)$*,*

a) and $6r + f \in B(K)$*, or*

b) an IPBD$(6r + f, f, K)$ *and* $6m + f \in B(K)$*,*

then there exists a PBD$(6mt + 6r + f, K)$ *containing flats of order* 7 *and a)* $6r + f$ *or b)* $6m + f$*.*

Proof. Truncate one group of the TD to size r to obtain a $\{t, t+1\}$-GDD $m^t r^1$. Give weight 6 to all points and apply Wilson's Fundamental Construction with ingredient GDDs 7-GDD 6^t and 7-GDD 6^{t+1} (Lemma 3.5) to get a 7-GDD $(6m)^t(6r)^1$. Apply Lemma 4.1 for the desired PBD. □

Lemma 4.14 *Let* $7 \in B(K)$*, suppose there is a TD*$(50, m)$ *and* $0 \le x, y \le m$*. If there exist an IPBD*$(6m + f, f, K)$*, and*

a) an IPBD$(6x + f, f, K)$ *and a PBD*$(6m + 42y + f, K)$*, or*

b) an IPBD$(6m + 42y + f, f, K)$ *and a PBD*$(6x + f, K)$*,*

then there exists a PBD$(294m + 6x + 42y + f, K)$*.*

Proof. Truncate one group of the TD$(50, m)$ to obtain a $\{49, 50\}$-GDD $m^{49}x^1$. Giving weight 48 to y points of one untruncated group and weight 6 to all other points Wilson's Fundamental Construction yields a 7-GDD $(6m)^{48}(6x)^1(6m + 42y)^1$ where we use as ingredient GDDs a 7-GDD 6^{49}, a 7-GDD 6^{50} (from Lemma 3.5), a 7-GDD $6^{48}48^1$ and a 7-GDD $6^{49}48^1$ (from Corollary 3.9). Applying Lemma 4.1 completes the proof. □

Lemma 4.15 *Let $7 \in B(K)$, suppose there is a TD$(50, m)$ and $0 \le x \le 1$, $0 \le y \le 3$, $0 \le z \le m - x - y$. If there exist an IPBD$(6m + f, f, K)$ and a PBD$(6x + 12y + 48z + f, f, K)$, then there exists a PBD$(294m + 6x + 12y + 48z + f, K)$.*

Proof. Truncate one group to size $x + y + z$ and give weight 48 to z, weight 12 to y and weight 6 to x points of the truncated group. All remaining points get weight 6. Now, Wilson's Fundamental Construction yields a 7-GDD $(6m)^{49}(6x + 12y + 48z)^1$ where we use as ingredient GDDs a 7-GDD 6^{49}, a 7-GDD 6^{50} (from Lemma 3.5), a 7-GDD $6^{49}12^1$ (from Lemma 3.6) and a 7-GDD $6^{49}48^1$ (from Corollary 3.9). Again, applying Lemma 4.1 completes the proof. □

Lemma 4.16 *Let $\{7, 13\} \subset B(K)$ and suppose there is a TD(n, m) with $51 \le n$. There is a PBD(v, K) with*

a) $v = 294m + 6t + 6r + 1$ if $0 \le r \le t \le m$ and $\{6m + 1, 6t + 1, 6r + 1\} \subset B(K)$, containing flats of order $7, 6m + 1, 6t + 1, 6r + 1$ and if $r > 0$ a flat of order 13;

b) $v = 294(m-1) + 6t + 6r + 1$ if $0 \le r < m$, $49 \le t < n$, $\{6m+1, 6r+1\} \subset B(K)$ and $6t + 1 \in B(\{7\})$, containing flats of order $7, 6m + 1, 6r + 1$ and if $r > 0, t > 49$ a flat of order 13;

c) $v = 294(m-1) + 6t + 6r - 5$ if $1 \le r \le m$, $50 \le t \le n$, $\{6m+1, 6r+1\} \subset B(K)$ and $6t + 1 \in B(\{7\})$, containing flats of order $7, 6m + 1, 6r + 1$ and if $r > 1, t > 50$ a flat of order 13;

d) $v = 294(m - 2) + 6t + 6r + 1$ if $49 \le r \le t \le n$, $6m + 1 \in B(K)$ and $\{6t + 1, 6r + 1\} \subset B(\{7\})$, containing flats of order $7, 6m + 1$ and if $r > 49$ a flat of order 13;

e) $v = 294(m - 2) + 6t + 6r - 5$ if $50 \le r \le t \le n$, $6m + 1 \in B(K)$ and $\{6t + 1, 6r + 1\} \subset B(\{7\})$, containing flats of order $7, 6m + 1$ and if $r > 50$ a flat of order 13;

f) $v = 300(m - 1) + 6t + 6r + 1$ if $0 \le r < 50$, $0 \le t < m$, $\{6m + 1, 6m - 5, 6t + 1\} \subset B(K)$ and $6r + 1 \in B(\{7\})$, containing flats of order $7, 6m - 5, 6t + 1$, and if $t > 0$ a flat of order 13, and if $r > 0$ a flat of order $6m + 1$;

g) $v = 300(m - 1) + 6t + 6r - 5$ if $1 \le r < 51$, $1 \le t \le m$, $\{6m + 1, 6m - 5, 6t + 1\} \subset B(K)$ and $6r + 1 \in B(\{7\})$, containing flats of order $7, 13, 6m - 5, 6t + 1$ and if $r > 1$ a flat of order $6m + 1$;

h) $v = 300(m-2)+6t+6r+1$ *if* $0 \le r < 50 \le t \le n$, $\{6m+1, 6m-5\} \subset B(K)$ *and* $\{6t+1, 6r+1\} \subset B(\{7\})$, *containing flats of order* $7, 6m-5$, *and if* $r > 0, t > 50$ *a flat of order* 13, *and if* $r > 0$ *a flat of order* $6m+1$;

i) $v = 300(m-2) + 6t + 6r - 5$ *if* $1 \le r < 51 \le t \le n$, $\{6m+1, 6m-5\} \subset B(K)$ *and* $\{6t+1, 6r+1\} \subset B(\{7\})$, *containing flats of order* $7, 13, 6m-5$ *and if* $r > 1$ *a flat of order* $6m+1$.

Proof. First note that a TD$(49, m)$, TD$(50, m)$ or TD$(51, m)$ is embedded in the TD(n, m). So, for a) truncate two groups of a TD$(51, m)$ to get a $\{49, 50, 51\}$-GDD $m^{49}t^1r^1$. Give weight 6 to all points in Wilson's Fundamental Construction and use the following ingredient GDDs, a 7-GDD 6^{49}, a 7-GDD 6^{50} from Lemma 3.5 and a $\{7, 13\}$-GDD 6^{51} from Lemma 3.6, to obtain a $\{7, 13\}$-GDD $(6m)^{49}(6t)^1(6r)^1$. Now adjoining an infinite point and filling groups gives the desired PBD. Note that the existence of a flat of order 13 is guaranteed only if there is a block of size 51 in the master GDD, i.e. only if $r, t > 0$.

The construction in the other cases is similar (always weight 6 for the points) so that we only need to find the master GDD in each case. Ingredient GDDs 7-GDD 6^t and 7-GDD 6^r exist by Lemma 3.5 if required.

For b/c) spike(=extend) one line of a TD$(50, m)$ to size t, then truncate a group to size r. If the t line and r group intersect in a deleted point we get b) a $\{49, 50, 51, t\}$-GDD $m^{49}r^1 1^{t-49}$, otherwise c) a $\{49, 50, 51, t\}$-GDD $m^{49}r^1 1^{t-50}$.

For d/e) we spike two lines of a TD$(49, m)$ to size t or r, respectively. Here we can assume the groups on the spiked lines coincide as much as possible. There are slight differences if spiked lines intersect within the TD or on the spikes. Within the TD we get d) a $\{49, 50, 51, r, t\}$-GDD $m^{49}2^{r-49}1^{t-r}$, while on the spikes e) a $\{49, 50, 51, r, t\}$-GDD $m^{49}2^{r-50}1^{t-r+1}$ is obtained.

For types f/g), we truncate a group of a TD$(51, m)$ to size t. Then we truncate a block to size r where we do not delete points from the truncated group. If we truncate a 50-block we get f) a $\{49, 50, 51, r\}$-GDD $m^r(m-1)^{50-r}t^1$ (so we need $t < m$ for a group truncation). Truncating a 51-block gives g) a $\{49, 50, 51, r\}$-GDD $m^{r-1}(m-1)^{51-r}t^1$ (so we need $0 < t$).

For types h/i), we spike one line of a TD$(50, m)$ to size t. Then we truncate a block to size r of the spiked TD where we do not delete points from the spike. Again, there are slight differences if we truncate a 50-block to obtain g) a $\{49, 50, 51, r, t\}$-GDD $m^r(m-1)^{50-r}1^{t-50}$ or a 51-block to get i) a $\{49, 50, 51, r, t\}$-GDD $m^{r-1}(m-1)^{51-r}1^{t-50}$ (so we need $51 \le t$ for a spike). □

If the master GDDs above do not have groups of order 7 or 13 we can successfully extend Lemma 4.16 by applying Lemma 4.1 on the master GDDs.

Lemma 4.17 *Let* $\{7, 13\} \subset B(K)$ *and suppose there is a* $TD(n, m)$ *with* $51 \leq n$.

a) If $0 \leq r \leq t \leq m$, *there is a* $PBD(6m + f, K)$ *containing a flat of order* f *and there are a* $PBD(6t+f, K)$ *and a* $PBD(6r+f, K)$ *of which at least one PBD contains a flat of order* f, *then a* $PBD(294m+6t+6r+f, K)$ *exists.*

f) If $0 \leq r < 50$, $0 \leq t < m$, *there are* $PBD(6m+f, K)$ *and* $PBD(6m-6+f, K)$ *containing flats of order* f, $6t + f \in B(K)$ *and* $6r + 1 \in B(\{7\})$, *then a* $PBD(300(m - 1) + 6t + 6r + f, K)$ *exists.*

g) If $1 \leq r < 51$, $1 \leq t \leq m$, *there are* $PBD(6m+f, K)$ *and* $PBD(6m-6+f, K)$ *containing flats of order* f, $6t + f \in B(K)$ *and* $6r + 1 \in B(\{7\})$, *then a* $PBD(300(m - 1) + 6t + 6r - 6 + f, K)$ *exists.*

Lemma 4.18 *Let* $\{7, 13\} \subset B(K)$ *and let* $q \geq 53$ *be a prime power. Suppose there are a* $PBD(6q + f, K)$ *and a* $PBD(6(q - 1) + f, K)$ *both with a flat of order* f. *Then there exists a* $PBD(306q - 6x + f, K)$ *for* $0 \leq x \leq 51$. *If in addition* $q \geq 101$ *and there is a* $PBD(6(q - 2) + f, K)$ *with a flat of order* f, *then there exists a* $PBD(306q - 6x - 12y + f, K)$ *for* $0 \leq x + y \leq 51$.

Proof. Consider an oval (or hyperoval if q is even) in a projective plane $\mathrm{PG}(2, q)$ and delete a point from the (hyper)oval. A $\mathrm{TD}(q+1, q)$ is obtained in which each group contains at most one point from the (hyper)oval. Deleting groups yields a $\mathrm{TD}(51, q)$ where all groups have exactly one (hyper)oval point. Delete x (hyper)oval points to get a $\{49, 50, 51\}$-GDD $q^{51-x}(q - 1)^x$ which in turn with Wilson's Fundamental Construction (weight 6 to all points, 7-GDD 6^{49}, 7-GDD 6^{50}, $\{7, 13\}$-GDD 6^{51}) provides a $\{7, 13\}$-GDD $(6q)^{51-x}(6(q-1))^x$. Finally, apply Lemma 4.1 to obtain the desired $\mathrm{PBD}(306q - 6x + f, K)$.

Now delete a non-tangent point (or just a non-hyperoval point if q is even) from $\mathrm{PG}(2, q)$ to obtain a $\mathrm{TD}(q + 1, q)$ in which there are exactly $(q + 1)/2$ groups containing no oval point and $(q + 1)/2$ groups containing exactly two oval points (or exactly $q/2$ groups containing no hyperoval point and $q/2 + 1$ groups containing exactly two hyperoval points if q is even). Since we require $q \geq 101$ we are able to delete groups in such a way that we get a $\mathrm{TD}(51, q)$ where all groups have exactly two (hyper)oval points. Now delete from x groups exactly one (hyper)oval point and from y groups two (hyper)oval points for a $\{49, 50, 51\}$-GDD $q^{51-x-y}(q - 1)^x(q - 2)^y$. Again, via Wilson's Fundamental Construction and Lemma 4.1 the desired $\mathrm{PBD}(306q - 6x - 12y + f, K)$ is obtained. □

We also need the *singular indirect product* construction which we take from [23].

	u	v	w	PBD with flat	a
1063	7	169	25	TD(7,24)+∞	20
1069	7	169	25	TD(7,24)+∞	19
1567	7	253	37	TD(7,36)+∞	34
1579	7	253	37	TD(7,36)+∞	32

Table 2: Proof of Corollary 4.20

Lemma 4.19 *If there is an IPBD*(v, w, K)*,* $u \in K$*,* $0 \leq a \leq w \leq v$*, the incomplete transversal design ITD*$(u; v-a, w-a)$ *exists, and* $u(w-a)+a \in B(K)$*, then there is a PBD*$(u(v-a)+a, K)$ *containing flats of order* u *and* $u(w-a)+a$*.*

As a first application of the singular indirect product we obtain the following.

Corollary 4.20 $1063, 1069, 1567$ *and* 1579 *are not essential in a Wilson basis for* $N_{1,6}$*.*

Proof. Use the previous lemma to show that $\{1063, 1069, 1567, 1579\} \subset B(\{7, 25, 37, 55, 61, 67\})$. The details are given in Table 2. The needed ITDs are all listed in [5]; they can also be constructed by Lemma 2.5 with $m = 16$ or 8. □

In order to apply the constructions above we need as many ingredient designs as possible. So we present in addition to the designs from Section 3 some new (resolvable) BIBDs for $k = 6, 12, 13$ by providing suitable base blocks.

Lemma 4.21 *If* $834 \leq t \leq 5460$*,* t *is even and* $q = 6t+1$ *is a prime power, then an RBIBD*$(30t+6, 6, 1)$ *exists and hence an IPBD*$(36t+7, 6t+1, \{7\})$*.*

Proof. As in [16] for $t \leq 832$ we construct $2t$ base blocks in $GF(5) \times GF(q)$. Let ω be a generator of the multiplicative group $GF(q)^*$ and $d = (q-1)/2$. We define blocks

$$S_0 = \{(0, \omega^0), (0, \omega^d), (1, \omega^{\gamma_1}), (1, \omega^{\gamma_1+d}), (4, \omega^{\gamma_2}), (4, \omega^{\gamma_2+d})\}$$

and

$$S_1 = \{(0, \omega^3), (0, \omega^{3+d}), (2, \omega^{\gamma_1+3}), (2, \omega^{\gamma_1+3+d}), (3, \omega^{\gamma_2+3}), (3, \omega^{\gamma_2+3+d})\}.$$

The base blocks are $S_{b,a} = (1, \omega^{6a}) \cdot S_b$ with $b = 0, 1$ and $a = 0, 1, \ldots, t-1$. It remains to specify γ_1, γ_2 such that the pure and mixed differences are evenly spread amongst the 6 cyclotomic classes. This is done in the appendix Table 8. Now with a new point ∞ adjoined and a new base block $\{\infty, (i, 0) : i = 0, 1, \ldots, 4\}$ an RBIBD$(30t + 6, 6, 1)$ is obtained where a resolution set is formed by the partial development of the base blocks $S_{b,a} \mod (5, -)$ augmented with the new base block. The development of this resolution set mod$(-, q)$ generates the other resolution sets. □

The necessary conditions for the existence of a BIBD$(v, 13, 1)$ are $v \equiv 1$ or 13 mod 156. Until now, only a few BIBD$(v, 13, 1)$ with v reasonably small were known to exist. The following BIBDs of order q, $q \equiv 1 \mod 156$ a prime power, are constructed by difference families using an approach described in [15].

Lemma 4.22 *BIBD*$(q, 13, 1)$ *exist for* $q \in A = \{6241, 8737, 9829, 14197, 15601, 16069, 16381, 16693, 18097, 19813, 20593, 20749, 21061, 21529, 21841, 21997, 22153, 22621, 22777, 23557, 23869, 24181, 24337, 25117, 25741, 26053, 26209, 26833, 27457, 28081, 28549, 29017, 29173, 29641, 30109, 30577, 31357, 31513, 31981\}$.

Proof. We construct base blocks in $GF(q)$. Let w be a cube root of unity and let $\{m, c, c', c''\} \subset GF(q)$. We define

$$S = \{0, 1, w, w^2, c, cw, cw^2, c', c'w, c'w^2, c'', c''w, c''w^2\}$$

and $S_i = m^i S$ with $i = 0, 1, \ldots, (q-1)/156$. With the values for m, w, c, c', c'' provided in the appendix Table 9 it is easy to check that every distance is covered exactly once by the blocks S_i. Note that if $q = p^n$ is a proper prime power, then an element $x \in GF(q), x = \sum_{i=0}^{n-1} a_i \omega^i$ is represented as $x = \sum_{i=0}^{n-1} a_i p^i$ where ω is a root of the primitive polynomial for q from Table 7. □

We can make the above difference families block disjoint by using Abel's adder.

Lemma 4.23 (Abel [1]) *Suppose* $q = k(k-1)t + 1$ *is a prime power and a* $(q, k, 1)$ *difference family over* $GF(q)$ *is given by* $B_i = m^i B_0$ *for* $i = 0, 1, \ldots, t-1$. *Then there is a block disjoint* $(q, k, 1)$ *difference family over* $GF(q)$.

Proof. Define a new difference family by $C_i = B_i + cm^i$. We will now show there is a choice of c such that C_i for $i = 0, 1, \ldots, t-1$ is the block disjoint difference family. Let $B_0 = (b_1, b_2, \ldots, b_k)$. Now $C_i = m^i(B_0 + c)$. Clearly, we cannot have $0 \in (B_0 + c)$, so this prohibits k values of c. If C_i and C_j have a common element and $i > j$, then $m^i b_y + cm^i = m^i b_z + cm^j$ for some $y \neq z$, i.e., $c = (b_z - m^{i-j} b_y)(m^{i-j} - 1)^{-1}$. This eliminates at most a further

$k(k-1)$ values of c for each value of $i-j$. So we have prohibited at most $q-(k-1)^2$ values of c, and there are at least $(k-1)^2$ values that work. $\square$

So we may again apply the Ray-Chaudhuri-Wilson construction to get RBIBDs.

Lemma 4.24 ([24]) *Let the set A be defined as in Lemma 4.22. If $q \in A$, then there is an RBIBD$(13q, 13, 1)$.*

The following family of 13-GDDs is particularly important as it gives the smallest PBDs in each fibre $19, 25, 31$ or 37 mod 42.

Lemma 4.25 *If $t = 1, 10, 13, 14, 19, 23, 30, 33, 44, 50, 51, 55, 59, 61, 63$ or $69 \leq t \leq 2730$, and $q = 12t+1$ is a prime power, then a 13-GDD 13^q exists and hence a PBD$(156t+13, \{13\})$.*

Proof. For $t=1$ just take a TD$(13,13)$. For $t=14$ replace the groups of a TD$(13,169)$ by the 13-GDD 13^{13} just obtained. For $t \neq 1, 14$ we construct t base blocks in $GF(13) \times GF(q)$. Let ω be a generator of the multiplicative group $GF(q)^*$. We define a block of size 13

$$S = \{(0,0)\} \cup \{(1, \pm\omega^0), (3, \pm\omega^{\gamma_1}), (4, \pm\omega^{\gamma_2}), (9, \pm\omega^{\gamma_3}), (10, \pm\omega^{\gamma_4}), (12, \pm\omega^{\gamma_5})\}.$$

The base blocks are $S_a = (1, \omega^{6a}) \cdot S$ with $a = 0, 1, \ldots, t-1$. With $\gamma_1, \ldots, \gamma_5$ from the appendix Table 10 we get a 13-GDD 13^q where the groups are $\{(i,x) : i = 0, 1, \ldots, 12\}$ for $x \in GF(q)$. If we consider the groups to be blocks of size 13 we obtain the desired PBD. $\square$

Moreover, we constructed block disjoint difference families of order q, $q \equiv 1$ mod 132 a prime power, with block size 12 which provide resolvable BIBDs of size 12 when we apply Ray-Chaudhuri-Wilson construction.

Lemma 4.26 *BIBD$(q, 12, 1)$ exist for $q \in B = \{5413, 6073, 6337, 6469, 6733, 6997, 7129, 7393, 7789, 7921, 8053, 8317, 8581, 8713, 9109, 9241, 9769, 9901, 10429, 10957, 11353, 11617, 11881, 12277, 12409, 12541, 13597, 13729, 14389, 14653, 15313, 15973, 16369, 16633, 17029, 17161, 17293, 18217, 18481, 19009, 19141, 19273, 19801, 20593, 20857, 21121, 21517, 21649, 22441, 22573, 23497, 23629, 23761, 23893, 24421, 25609, 25741, 25873, 27061, 27457, 28513, 28909, 29173, 29437, 29569, 29833, 30097, 30493, 30757, 31153, 32077, 32341\}$. If $q \in B$, then there is a BIBD$(q, 12, 1)$, an RBIBD$(12q, 12, 1)$ and an IPBD$(12q + (12q-1)/11, (12q-1)/11, \{13\})$.*

Proof. We construct base blocks in $GF(q)$. Let w be a cube root of unity and let $\{m, c, c', c''\} \subset GF(q)$. We define

$$S = \{1, w, w^2, c, cw, cw^2, c', c'w, c'w^2, c'', c''w, c''w^2\}$$

and $S_i = m^i S$ with $i = 0, 1, \dots, (q-1)/132$. With the values for m, w, c, c', c'' provided in the appendix Tables 11 and 12 it is easy to check that every distance is covered exactly once by the blocks S_i. $\square$

Lemma 4.27 *If $t \notin \{30, 33\}$, $29 \le t \le 2730$, and $q = 12t + 1$ is a prime power, then an RBIBD$(132t + 12, 12, 1)$ exists and hence an IPBD$(144t + 13, 12t + 1, \{13\})$.*

Proof. Here, we construct t base blocks in $GF(11) \times GF(q)$. Let ω be a generator of the multiplicative group $GF(q)^*$. We define a block of size 12

$$S = \{(0, \pm\omega^0), (1, \pm\omega^{\gamma_1}), (3, \pm\omega^{\gamma_2}), (4, \pm\omega^{\gamma_3}), (5, \pm\omega^{\gamma_4}), (9, \pm\omega^{\gamma_5})\}.$$

The base blocks are $S_a = (1, \omega^{6a}) \cdot S$ with $a = 0, 1, \dots, t-1$. With $\gamma_1, \dots, \gamma_5$ from the appendix Table 13 and the additional base block $\{\infty, (i, 0) : i = 0, 1, \dots, 10\}$ an RBIBD$(132t + 12, 12, 1)$ is obtained. $\square$

We actually tried all prime powers < 32768 in the appropriate residue class for our direct constructions. In Lemma 4.22, we had failures for $q = 156t + 1$ with $q \in$ {157, 313, 625, 937, 1093, 1249, 1873, 2029, 2341, 2809, 3121, 3433, 4057, 4993, 6397, 6553, 6709, 7177, 7333, 7489, 8269, 8581, 8893, 9049, 10141, 10453, 10609, 11701, 12637, 13417, 13729, 14821, 15289, 15913, 17161, 17317, 18253, 19501, 24649, 28393, 32761}. Thus we failed in 41 of the 80 possible cases. Greig [15] also looked at cosets of size 13, with only success for $q = 21061$ (which adds nothing new here). We will get a quick failure if $\log(x^{52t} - 1) \equiv 0 \bmod 26$, and we had 5 quick failures. In Lemma 4.26, we had failures for $q = 132t + 1$ with $q \in$ {397, 529, 661, 1321, 1453, 1849, 2113, 2377, 3037, 3169, 3301, 3433, 3697, 4093, 4357, 4489, 4621, 5281}. Thus we failed in 18 of the 90 possible cases. In Lemma 4.27, we had failures for $q = 12t + 1$ with $t \in$ {1–6, 8–10, 13–16, 19, 20, 23, 24, 26, 28, 30, 33}. In Lemma 4.25, we had failures for $q = 12t + 1$ with $t \in$ {2–6, 8, 9, 15, 16, 20, 24, 26, 28, 31, 34–36, 38, 45, 48, 52, 56, 64}.

5 PBD-Closure of $K = \{7, 13\}$

In this section, we show that all positive integers $v \equiv 1 \bmod 6$ are in $B(\{7, 13\})$ with the possible exceptions in $Q_{\{7,13\}}$. $Q_{\{7,13\}}$ contains 3960 elements which are listed in the appendix (Table 14). To reduce the problem to a finite one we first construct a representative PBD$(v, \{7, 13\})$ in each possible residue class r modulo $(14 \cdot 84)$, $r \not\equiv 1, 7 \bmod 42$.

Lemma 5.1 *Let $R = \{r_{i,j} : i = 0, 1, \dots, 13, j = 13, 19, \dots, 79\}$ be given by the entries in Table 3. If $r \in R$, then $r \in B(\{7, 13\})$.*

13	19	25	31	37	55	61	67	73	79
13	12955	12961	21199	21205	3583	14173	24763	3601	3607
2449	13039	27157	21283	9529	4843	18961	18967	7213	18979
3709	15475	10777	21367	19021	3751	16693	17875	14353	13183
2617	16735	16741	19099	4993	307	14425	16783	15613	19147
3877	14467	6241	21535	19189	8623	1573	18043	14521	12175
8665	13375	21613	19267	5161	8707	15769	19303	13429	19315
517	14635	18169	19351	17005	559	14677	17035	19393	19399
601	15895	19429	19435	2977	4171	12409	15943	19477	19483
5389	12451	19513	19519	17173	8959	12493	17203	7801	3103
4297	12535	16069	29011	13729	4339	11401	24343	21997	23179
2029	13795	2041	29095	14989	2071	17365	16195	7969	12679
2113	20935	16237	28003	13897	5683	9217	20983	6877	20995
2197	13963	15145	24559	21037	6943	8125	16363	21073	17551
1105	15223	16405	24643	9361	1147	21145	16447	4693	21163

Table 3: Representative PBD($r_{i,j}, \{7, 13\}$) with $r_{i,j} \equiv 84i + j \bmod 1176$

Proof. We give for each entry from the table a construction and the parameters used. All requisite (I)PBDs are easily obtained from Lemmas 3.1, 3.3, 4.2 or are constructed within this proof. Orders which do not occur in the table but are required as components are marked with an asterisk.

An IPBD of order 3895* with a hole of size 649 exists by Lemma 3.2 with $t = 18$. The entries 3103, 3607, 12175, 12679, 13183 are from Lemma 3.3 with $t = 86$, 100, 338, 352, 366. Orders 4993, 7801, 9361, 13729, 14353, 17005, 17551, 19189, 19267 are constructed by Lemma 4.2 with $k = 13$, $n = 384$, 600, 720, 1056, 1104, 1308, 1350, 1476, 1482. A PBD of order 3703* with a hole of size 517 exists by Corollary 4.5 with $m = 43$, $t = 15$. Moreover, 2401*, 2449 are obtained from Lemma 4.10 with $n = 7$, $m = 50, 51$ and 8707 comes from Lemma 4.11 with $m = 97$, $t = 91$, $f = 13$. 4339, 8665 are from Lemma 4.12 with $m = 49$, $f = 7$ and $m = 97$, $f = 13$. 16447, 28003 are from Lemma 4.15 with $m = 49$, $x = 0$, $y = 2$, $z = 42$, $f = 1$ and $m = 83$, $x = 0$, $y = 3$, $z = 74$, $f = 13$. From Lemma 4.17.a we get 24559, 24643 with $m = 83$, $r = 12$, $t = 12, 26$, $f = 13$ and 29011, 29095 with $m = 97$, $r = 12, 26$, $t = 68$, $f = 13$. Furthermore, 19303, 19315, 19351, 19393, 19399, 19429, 19435, 19477, 19483, 19513, 19519, 21535, 21613 are from Lemma 4.18 with $q = 64$, $f = 1$, $47 \geq x \geq 11$ or $q = 71$, $f = 1$, $x = 32, 19$. The entries 6241, 16069, 16693, 21997 are taken from Lemma 4.22. Moreover, 1573, 2041, 2197, 2977, 3601, 4693, 5161, 6877, 7969, 9217, 9529, 10777, 11401, 12493, 12961, 13429, 13897, 14521, 14677, 14989, 15613, 15769, 16237, 17173, 21073, 27157 are from Lemma 4.25 with $t = 10, 13, 14, 19, 23, 30, 33, 44, 51, 59, 61, 69, 73, 80, 83, 86, 89, 93, 94, 96, 100, 101, 104, 110, 135, 174$. 7213 is from Lemma 4.27 with $t = 50$. Finally, all remaining orders r can be constructed using Lemma 4.19 (Singular Indirect Product) or Lemma 4.16. The parameters of these constructions are presented in Table 4 and Table 5. The needed ITDs can all be constructed by Lemmas 2.5–2.9. □

Table 4: Applications of the Singular Indirect Product

r	u	v	w	a	r	u	v	w	a
307	7	49	7	6	517	7	85	13	13
559	7	85	7	6	601	7	91	7	6
1105	7	169	13	13	1147	7	169	7	6
1387*	7	223	37	29	1471*	7	223	37	15
1483*	7	217	7	6	2029	7	295	7	6
2071	7	301	7	6	2113	7	307	7	6
2437*	7	367	61	22	2617	7	379	7	6
3583	7	517	7	6	3709	7	583	97	62
3751	13	295	7	7	3877	7	559	7	6
4171	7	601	7	6	4297	7	631	91	20
4843	7	727	121	41	5389	7	871	145	118
5683	7	871	145	69	6943	7	1087	181	111
8125	13	637	13	13	8623	7	1375	229	167
8959	7	1447	241	195	12409	7	1807	259	40
12451	7	1813	259	40	12535	7	1879	313	103
12955	7	1951	325	117	13039	13	1015	13	13
13375	7	2023	337	131	13795	7	2095	349	145
13963	7	2065	295	82	14173	7	2101	301	89
14425	7	2143	307	96	14467	7	2149	307	96
14635	7	2239	373	173	15145	13	1177	13	13
15223	13	1183	13	13	15475	7	2383	397	201
15895	7	2455	409	215	15943	7	2383	397	123
16195	7	2359	337	53	16363	7	2455	409	137
16405	7	2395	343	60	16735	7	2599	433	243
16741	7	2401	301	11	16783	7	2527	421	151
17035	7	2437	295	4	17203	7	2599	433	165
17365	13	1513	217	192	17875	13	1387	13	13
18043	7	2743	457	193	18169	7	2689	385	109
18967	13	1471	13	13	20983	7	3247	541	291
23179	7	3463	577	177	24343	7	3703	517	263
24763	7	3895	649	417					

Table 5: Applications of Lemma 4.16

r	Case	n	m	t	r	r	Case	n	m	t	r
18961	f)	65	64	2	8	18979	i)	65	64	56	8
19021	c)	65	64	56	28	19099	i)	65	64	56	28
19147	g)	65	64	14	28	20935	a)	51	71	2	8
20995	c)	72	71	56	14	21037	c)	72	71	63	14
21145	a)	51	71	2	43	21163	i)	72	71	63	15
21199	i)	72	71	56	28	21205	c)	72	71	56	49
21283	i)	72	71	70	28	21367	i)	72	71	63	49

Lemma 5.2 *Let* $m \equiv 1$ *mod* 14*,* $3501 \le m \le 4901$*. If* $v = 12m + 1$*, then* $v \in B(\{7, 13\})$*.*

Proof. This follows by application of Lemma 4.14 with parameters $m = 139$, $x = 50$, $y = 20, 24, \ldots, 136$, $f = 7$; $m = 139$, $x = 99$, $y = 133, 137$, $f = 7$; $m = 167$, $x = 50$, $y = 0, 4, \ldots, 164$, $f = 7$ and $m = 181$, $x = 50$, $y = 2, 6, \ldots, 126$, $f = 7$ which gives v in the intervals $42013 - 46885$, $47053 - 47221$, $49405 - 56293$ and $53605 - 58813$. The existence of all requisite PBDs is ensured by Lemma 3.1. The remaining values from the interval $47389 - 49237$ are obtained from Corollary 4.5 with parameters $m = 539$, $t = 352, 408, 436, 492$; $m = 540$, $t = 534$; $m = 560$, $t = 86, 338, 366$; $m = 574$, $t = 86$; Corollary 4.7 with $u = 85$, $m = 547$, $t = 317, 401$ and Lemma 4.25 with $t = 308$. Here, the requisite PBDs exist by Lemma 3.1, Lemma 5.1, Lemma 4.2 ($k = 7$, $n = 463$), or Lemma 4.19 ($u = 7$, $v = 421$, $w = 7$, $a = 0$; $u = 7$, $v = 463$, $w = 7$, $a = 6$; $u = 7$, $v = 931$, $w = 7$, $a = 6$). □

Lemma 5.3 *If* $v \equiv 1$ *mod* 6*,* $v \ge 319825$ *then* $v \in B(\{7, 13\})$*.*

Proof. Note $v \equiv 1, 7 \bmod 42$ is covered by Lemma 3.1. For $v \equiv 13, 19, 25, 31, 37 \bmod 42$, there is exactly one $r_{i,j}$ from Table 3 such that $v \equiv 7 \cdot 84 + r_{i,j} \bmod 1176$. If $v \ge 299628 + r_{i,j}$, then there is a unique representation $v = 84m + r_{i,j}$ such that $m \equiv 7 \bmod 14$, $m \ge 3567$. So a TD$(14, m)$ exists by Lemma 2.3 and both a BIBD$(6m+1, 7, 1)$ and a BIBD$(12m+1, 7, 1)$ exist by Lemma 3.1. Thus, by applying Corollary 4.5 with $r_{i,j} = 6t+1$ we can construct every value v exceeding 319819 except those with $v \equiv r \bmod 1176$ where $r \in E = \{199, 235, 241, 247, 283, 367, 451, 535, 655, 697, 955, 1033\}$. The exceptions occur if $t > m$ that is $r_{i,j} > 21403$. In order to apply Corollary 4.5 we need in these cases that $v \ge 15(r_{i,j} - 1) + 1$. Noting that $\max\{r_{i,j}\} = 29095$ we can construct in the exceptional cases every value exceeding 435235.

So it remains to consider the interval $319819 < v \leq 435235$ where $v \equiv r \bmod 1176$, with $r \in E$. Write $v = 84m + r_{i,j}$ such that $m \equiv 1 \bmod 14$, $m \geq 3567$. Again a $\mathrm{TD}(14, m)$ exists by Lemma 2.3, a $\mathrm{BIBD}(6m + 1, 7, 1)$ exists by Lemma 3.1 and a $\mathrm{PBD}(12m + 1, \{7, 13\})$ exists by Lemma 5.2. Hence, if $r_{i,j} \leq 21403$ we can apply Corollary 4.5 to get a $\mathrm{PBD}(v, \{7, 13\})$. This works except for $v \equiv 451, 955 \bmod 1176$.

Now, let $v \equiv 451, 955 \bmod 1176$. There is a representation $v = 84m + r_{4,31} = 84m + 21535$ such that $m \equiv 1, 7 \bmod 14$. If $v \geq 323011$, then we can construct a $\mathrm{PBD}(v, \{7, 13\})$ using Corollary 4.5. We are done if we can give a construction for $v = 320323, 320827, 321499, 322003, 322675$: apply Corollary 4.5 with $(m, t) = (3581, 3253)$, $(3588, 3239)$, $(3574, 3547)$, $(3581, 3533)$, $(3588, 3547)$. The requisite PBDs of order $6m + 1$ are obtained from Lemma 4.18 with $q = 71$, $x = 33, 40, 47$, $f = 1$, those of order $12m + 1$ are obtained from Corollary 4.5 with $m = 483$, $t = 386, 400, 414$ and the one of order $6t + 1$ are constructed in Lemma 5.1. □

The next result is obtained with a computer run in which we used all constructions and previously known designs mentioned above to eliminate possible exceptions $v \leq 319819$. There is not enough space to write down all 49369 constructions here, but we provide a web-page
`ftp://ftp.math.uni-rostock.de/pub/members/mgruttm/pbdclosure7_13/index.html`
where for each v at least one construction is given. The computer search left over a set $Q_{\{7,13\}}$ of 3960 possible exceptions listed in Table 14. The largest possible exception is 98683.

Lemma 5.4 *If $v \equiv 1$ mod 6, $v \leq 319819$, $v \notin Q_{\{7,13\}}$, then $v \in B(\{7, 13\})$.*

Now, our main result follows from Lemma 5.3 and Lemma 5.4.

Theorem 5.5 *If $v \equiv 1$ mod 6, $v \notin Q_{\{7,13\}}$, then $v \in B(\{7, 13\})$.*

6 PBD-Closure of sets K where $\{7, 13\} \subset K \subseteq \{7, 13, 19, 25, 31, 37, 43\}$

Using the same methods as in the previous section we determined the PBD-closure of all sets K where $\{7, 13\} \subset K \subseteq \{7, 13, 19, 25, 31, 37, 43\}$ leaving in each case a number of possible exceptions. The largest exceptions for each K and each fibre $6t + 1$ modulo 42 are represented in Table 6. Again, there is not enough space for the constructions and the sets of exceptions. Hence, we give details at
`ftp://ftp.math.uni-rostock.de/pub/members/mgruttm/pbdclosure7_13/index.html`.

Table 6: Largest possible exception in each residue class $6t + 1$ modulo 42 for all sets K with $\{7, 13\} \subseteq K \subseteq \{7, 13, 19, 25, 31, 37, 43\}$

$6t + 1$ modulo 42 K	1	7	13	19	25	31	37
$\{7, 13\}$	2605	1645	14293	82549	98683	91507	88447
$\{7, 13, 19\}$	2605	805	13915	13081	26191	90751	18811
$\{7, 13, 25\}$	2605	1645	12277	77635	12247	22417	33133
$\{7, 13, 19, 25\}$	2605	805	12193	11065	12247	18763	18685
$\{7, 13, 31\}$	2605	1645	12277	18541	24721	11833	48631
$\{7, 13, 19, 31\}$	2605	805	12193	11065	18463	11077	18055
$\{7, 13, 25, 31\}$	2605	1645	12277	18541	11071	11833	23305
$\{7, 13, 19, 25, 31\}$	2605	805	12193	11065	11071	9859	17635
$\{7, 13, 37\}$	1975	1435	13075	48739	61807	64627	13099
$\{7, 13, 19, 37\}$	1975	805	13075	10099	18757	36991	13099
$\{7, 13, 25, 37\}$	1975	1435	9799	23917	9811	18217	10999
$\{7, 13, 19, 25, 37\}$	1975	805	8413	9511	9811	17923	6631
$\{7, 13, 31, 37\}$	1975	1435	9799	18541	24385	10111	10117
$\{7, 13, 19, 31, 37\}$	1975	805	9799	10099	18379	10111	10117
$\{7, 13, 25, 31, 37\}$	1975	1435	9799	17995	7417	9817	7765
$\{7, 13, 19, 25, 31, 37\}$	1975	805	5935	6613	6619	5995	6631
$\{7, 13, 43\}$	2605	1645	14293	82549	95785	89575	82567
$\{7, 13, 19, 43\}$	2605	805	13915	13081	26191	79159	18685
$\{7, 13, 25, 43\}$	2605	1645	12277	77635	12247	22417	25447
$\{7, 13, 19, 25, 43\}$	2605	805	12193	11065	12247	18763	18685
$\{7, 13, 31, 43\}$	2605	1645	12277	18541	24721	11833	48631
$\{7, 13, 19, 31, 43\}$	2605	805	12193	11065	18463	11077	17803
$\{7, 13, 25, 31, 43\}$	2605	1645	12277	17281	11071	11833	23305
$\{7, 13, 19, 25, 31, 43\}$	2605	805	12193	11065	11071	9859	17635
$\{7, 13, 37, 43\}$	799	1435	13075	48739	60379	64627	13099
$\{7, 13, 19, 37, 43\}$	799	805	13075	10099	18757	36991	13099
$\{7, 13, 25, 37, 43\}$	799	1435	9799	23161	9811	18217	10999
$\{7, 13, 19, 25, 37, 43\}$	799	805	8413	9511	9811	17923	6631
$\{7, 13, 31, 37, 43\}$	799	1435	9799	18541	24385	10111	10117
$\{7, 13, 19, 31, 37, 43\}$	799	805	9799	10099	18379	9523	10117
$\{7, 13, 25, 31, 37, 43\}$	799	1435	9799	17281	7417	9817	7765
$\{7, 13, 19, 25, 31, 37, 43\}$	799	805	5935	6613	6619	5995	6631

7 Acknowledgements

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A Appendix

The first table gives the primitive polynomials for all proper prime powers 1 mod 12 which are $\leq 2^{15}$. The next six tables give the parameters required in the proof of Lemmas 4.21, 4.22, 4.25. 4.26 (two tables) and 4.27. The entry * indicates that the generating element ω is a root of the primitive polynomial in the first table. Finally in the last table, we list all integers v for which the existence of a PBD($v, \{7, 13\}$) is not known.

Table 7: Table of primitive polynomials of $GF(p^n)$ with $p^n \equiv 1 \bmod 12$, $f(x) = b_0x^0 + b_1x^1 + \ldots + b_{n-1}x^{n-1} + x^n$

p^n	p	n	$b_0, \ldots, b_{n-1}$	p^n	p	n	$b_0, \ldots, b_{n-1}$	p^n	p	n	$b_0, \ldots, b_{n-1}$
25	5	2	2,1	625	5	4	3,1,0,1	15625	5	6	2,0,0,0,0,1
49	7	2	3,1	2401	7	4	3,0,1,1	121	11	2	7,1
14641	11	4	8,0,0,1	169	13	2	2,1	2197	13	3	2,0,1
28561	13	4	6,0,1,1	289	17	2	3,1	361	19	2	2,1
529	23	2	7,1	841	29	2	3,1	961	31	2	12,1
1369	37	2	5,1	1681	41	2	12,1	1849	43	2	3,1
2209	47	2	13,1	2809	53	2	5,1	3481	59	2	2,1
3721	61	2	2,1	4489	67	2	12,1	5041	71	2	11,1
5329	73	2	11,1	6241	79	2	3,1	6889	83	2	2,1
7921	89	2	6,1	9409	97	2	5,1	10201	101	2	3,1
10609	103	2	5,1	11449	107	2	5,1	11881	109	2	6,1
12769	113	2	10,1	16129	127	2	3,1	17161	131	2	14,1
18769	137	2	6,1	19321	139	2	2,1	22201	149	2	3,1
22801	151	2	12,1	24649	157	2	6,1	26569	163	2	11,1
27889	167	2	5,1	29929	173	2	5,1	32041	179	2	7,1
32761	181	2	18,1								

Table 8: Table of γ_1, γ_2 for RBIBD($5q + 1, 6, 1$) construction

q	ω	γ_1, γ_2	q	ω	γ_1, γ_2	q	ω	γ_1, γ_2	q	ω	γ_1, γ_2
5041	*	139, 2672									
5077	2	2, 4951	5101	6	4912, 4634	5113	19	3542, 4393	5197	7	1330, 395
5209	17	2102, 2857	5233	10	2552, 1021	5281	7	4898, 3922	5329	*	1184, 1846
5413	5	3052, 2318	5437	5	116, 349	5449	7	760, 4034	5521	11	3692, 5446
5557	2	1, 4127	5569	13	4372, 5024	5581	6	586, 4997	5641	14	3910, 2897
5653	5	2926, 2486	5689	11	136, 4397	5701	2	1, 5456	5737	5	1777, 1817
5749	2	1, 3920	5821	6	3136, 1442	5857	7	4546, 1934	5869	2	1, 4907
5881	31	1108, 128	5953	7	1364, 1807	6037	5	266, 3574	6073	10	3622, 3260
6121	7	124, 5162	6133	5	3518, 2134	6217	5	1, 3365	6229	2	1, 2813
6241	*	320, 6106	6277	2	1, 662	6301	10	5722, 5495	6337	10	418, 4532
6361	19	4796, 3673	6373	2	1, 3776	6397	2	1, 5075	6421	6	760, 4985
6469	2	1, 1790	6481	7	2272, 1670	6529	7	5312, 4423	6553	10	392, 6151
6577	5	974, 1021	6637	2	2, 6337	6661	6	5164, 6098	6673	5	1, 4238
6709	2	2, 5383	6733	2	1, 32	6781	2	1, 6284	6793	10	2428, 782
6829	2	1, 4112	6841	22	5764, 1853	6889	*	1, 3449	6949	2	1, 6668
6961	13	6320, 2368	6997	5	2500, 2939	7057	5	1, 6080	7069	2	6493, 6512
7129	7	4108, 533	7177	10	1964, 4900	7213	5	1, 1289	7237	2	1, 7118
7297	5	2054, 3430	7309	6	4907, 5908	7321	7	5764, 6206	7333	6	3196, 7325
7369	7	1316, 535	7393	5	5768, 2344	7417	5	1490, 2236	7477	2	1, 3056
7489	7	1520, 2197	7537	7	1706, 1486	7549	2	1, 6839	7561	13	6556, 4190
7573	2	2, 6448	7621	2	1, 5738	7669	2	2, 1549	7681	17	3934, 89
7717	2	2, 4930	7741	7	1712, 3109	7753	10	1921, 6581	7789	2	1, 1610
7873	5	1, 4520	7921	*	1, 4049	7933	2	1, 6203	7993	5	6080, 7828
8017	5	1762, 5627	8053	2	2, 7291	8089	17	604, 2750	8101	6	3970, 32
8161	7	6454, 1721	8209	7	4732, 7640	8221	2	2, 7999	8233	10	6032, 2938
8269	2	1, 4148	8293	2	1, 2564	8317	6	406, 2831	8329	7	4246, 5768
8353	5	1, 155	8377	5	199, 4220	8389	6	3154, 2429	8461	6	1490, 7615
8521	13	5282, 4522	8581	6	7358, 7180	8629	6	5146, 2231	8641	17	6502, 5798
8677	2	2, 4462	8689	13	8086, 3332	8713	5	1, 4457	8737	5	4924, 7364
8761	23	3862, 8291	8821	2	1, 7016	8893	5	1, 296	8929	11	4030, 6302
8941	6	934, 3845	9001	7	550, 5153	9013	5	5744, 7588	9049	7	4762, 5168
9109	10	1294, 5339	9133	6	7312, 5465	9157	6	3038, 7204	9181	2	2, 1078
9241	13	1724, 6877	9277	5	1, 2627	9337	5	3788, 664	9349	2	1, 2396
9397	2	1, 8720	9409	*	3332, 4939	9421	2	2, 2899	9433	5	9386, 9103
9601	13	4052, 8371	9613	2	2, 8749	9649	7	5240, 7084	9661	2	1, 8888
9697	10	2908, 911	9721	7	7846, 872	9733	2	1, 5981	9769	13	1600, 6401
9781	6	1306, 470	9817	5	9091, 3092	9829	10	8407, 8687	9901	2	2, 7246
9949	2	2, 3502	9973	11	9905, 6202	10009	11	7316, 1003	10069	2	1, 7385
10093	2	1, 4121	10141	2	1, 5819	10177	7	7840, 7553	10201	*	1, 5201
10273	10	7220, 6955	10321	7	6868, 9698	10333	5	6808, 5699	10357	2	2, 8767
10369	13	1288, 4556	10429	7	5230, 4388	10453	5	325, 2	10477	2	2, 9613

Table 8: Table of γ_1, γ_2 for RBIBD$(5q+1, 6, 1)$ construction (cont.)

q	ω	γ_1, γ_2	q	ω	γ_1, γ_2	q	ω	γ_1, γ_2	q	ω	γ_1, γ_2
10501	2	1, 1781	10513	7	10300, 4175	10597	5	1, 8246	10609	*	9152, 3670
10657	7	832, 3224	10729	7	10004, 7402	10753	11	1526, 4051	10789	2	1, 10187
10837	2	1, 3503	10861	2	1, 3776	10909	2	1, 8576	10957	5	1, 5375
10993	7	8420, 7621	11113	13	10982, 10129	11149	10	3316, 5636	11161	7	550, 1244
11173	5	1, 4388	11197	2	2, 8641	11257	10	3272, 3517	11317	2	1, 10364
11329	7	6988, 5372	11353	7	10726, 9839	11437	2	1, 6539	11449	*	1, 6374
11497	7	830, 4138	11593	5	6526, 1427	11617	10	6992, 9850	11677	2	2, 3949
11689	7	9446, 9679	11701	6	2165, 7399	11821	2	2023, 2054	11833	5	8684, 5908
11881	*	8360, 8950	11941	10	5575, 10760	11953	5	1, 8366	12037	5	3043, 659
12049	13	10048, 4532	12073	7	1480, 3446	12097	5	4756, 11165	12109	6	1156, 9107
12157	2	1, 4187	12241	7	1994, 9940	12253	2	1, 4187	12277	2	1, 10226
12289	11	4172, 2470	12301	2	1, 10916	12373	2	1, 1193	12409	7	7448, 10492
12421	7	3604, 10133	12433	13	6362, 8014	12457	10	3400, 7106	12517	6	2404, 5555
12541	14	4735, 8225	12553	5	7852, 7853	12577	10	4880, 12298	12589	2	1, 10634
12601	11	10718, 2314	12613	2	1, 12092	12637	2	1, 1064	12697	7	2468, 2143
12721	13	1570, 7628	12757	2	2, 7567	12769	*	1, 8783	12781	2	1, 1100
12829	2	1, 662	12841	21	9590, 418	12853	5	1, 5402	12889	13	9982, 8822
12973	14	1, 2519	13009	7	2582, 9454	13033	5	5006, 9103	13093	6	7162, 5744
13177	5	1, 3584	13249	7	11582, 11584	13297	5	12055, 10058	13309	6	12670, 4796
13381	10	7855, 4640	13417	5	10640, 7327	13441	11	8728, 4601	13477	2	2, 9088
13513	5	6772, 3707	13537	7	9542, 7150	13597	5	2756, 2170	13633	5	12112, 12539
13669	6	1, 335	13681	22	12352, 11822	13693	6	6652, 1856	13729	23	13240, 9743
13789	7	10700, 11728	13873	5	3838, 11099	13921	7	316, 1814	13933	2	1, 5
14029	6	766, 6005	14149	6	11944, 10304	14173	2	2, 3319	14197	11	12994, 3098
14221	2	7951, 8267	14281	19	12674, 4423	14293	6	1468, 10805	14341	2	2, 3193
14389	2	1, 2672	14401	11	10264, 10265	14437	5	10028, 1018	14449	22	3956, 1
14461	2	1, 14270	14533	2	2, 1957	14557	2	2, 8212	14593	5	1, 10751
14629	2	13699, 1328	14641	*	1, 5909	14653	2	1, 10595	14713	5	13996, 11606
14737	10	1588, 13580	14797	2	1, 263	14821	2	2, 7120	14869	2	1, 8693
14929	7	11116, 6950	15013	2	1, 4772	15061	2	1, 5276	15073	5	1, 1718
15121	11	952, 953	15193	5	1, 13289	15217	10	12508, 13388	15241	11	11120, 11860
15277	6	14104, 2312	15289	11	2422, 2423	15313	5	13384, 3734	15349	2	1, 9620
15361	7	2696, 10456	15373	2	1, 3191	15493	5	682, 11783	15541	6	11810, 11059
15601	23	14680, 1067	15625	*	1, 2966	15649	11	2720, 13648	15661	2	1, 6683
15733	6	12778, 5399	15817	5	5908, 8213	15877	5	10826, 13150	15889	21	1046, 11086
15901	10	7934, 15874	15913	5	10399, 15884	15937	7	6034, 14057	15973	7	6202, 15356
16033	5	11842, 12152	16057	7	14072, 13192	16069	2	1, 11744	16129	*	9344, 14698
16141	6	8495, 2323	16189	2	1, 7151	16249	17	1552, 617	16273	7	4628, 14185
16333	2	1, 2051	16369	7	2384, 7294	16381	2	1, 2777	16417	10	12104, 10915
16453	2	1, 10799	16477	2	2, 9283	16561	7	14656, 6188	16573	2	2, 6043
16633	15	5080, 8360	16657	5	9163, 5075	16693	2	2, 11515	16729	13	4898, 10468
16741	6	8080, 5459	16921	17	12920, 14473	16981	2	1, 12800	16993	10	1474, 11753
17029	10	11056, 8372	17041	7	11060, 14719	17053	2	2, 3835	17077	2	1, 12101
17137	5	16760, 12640	17161	*	1, 11225	17209	14	12448, 4949	17257	5	7484, 7450
17293	7	13666, 13463	17317	2	2, 13246	17341	6	11932, 12620	17377	7	8428, 7226
17389	2	1, 2498	17401	11	2228, 5983	17449	14	15778, 2732	17497	5	13184, 5302
17509	2	1, 8018	17569	11	15652, 16250	17581	10	3955, 10442	17713	7	15436, 12746
17737	7	4792, 2837	17749	2	2, 2785	17761	19	17426, 2638	17881	7	2056, 3233
17929	11	16508, 2962	17977	5	2812, 4379	17989	2	1, 998	18013	2	1, 9518
18049	13	10544, 12631	18061	6	2146, 5492	18097	5	14114, 2890	18121	23	4336, 13016
18133	5	1, 12989	18169	11	3140, 6952	18181	2	1, 1661	18217	7	14410, 15248
18229	2	2, 4819	18253	5	14860, 12986	18289	13	14810, 6064	18301	6	1462, 8693
18313	10	15884, 7108	18397	6	3568, 3569	18433	5	6296, 826	18457	5	9572, 6664
18481	13	3824, 1474	18493	2	2, 10612	18517	6	16918, 18227	18541	6	10774, 10541
18553	5	15734, 17341	18637	2	2, 10984	18661	10	2140, 455	18757	2	1, 3233
18769	*	1, 6425	18793	5	1, 12797	18913	7	9290, 10456	18973	2	1, 18872
19009	23	9100, 17702	19069	2	1, 13226	19081	17	10144, 13082	19141	2	1, 15572
19213	5	16090, 6335	19237	2	1, 18398	19249	7	5728, 7874	19273	5	18127, 9122
19309	6	3886, 2162	19321	*	140, 17155	19333	2	1, 13478	19381	7	5315, 5989
19417	5	1, 197	19429	6	11350, 10025	19441	13	4264, 17684	19477	6	12616, 13475
19489	19	15676, 1226	19501	2	1, 18218	19597	2	1, 5942	19609	13	3680, 16945
19681	11	7420, 1004	19717	2	1, 11135	19753	5	1, 11984	19777	11	1, 6197
19801	13	844, 15794	19813	2	2, 7	19861	11	2972, 5146	19993	10	9028, 7298
20029	2	1, 2834	20089	7	9188, 10291	20101	6	3382, 13340	20113	10	11512, 5825
20149	2	2209, 4421	20161	13	19286, 4951	20173	2	2, 13003	20233	5	956, 9229
20269	2	2, 14377	20341	2	1, 11054	20353	5	15032, 11791	20389	6	5428, 9485
20509	2	1, 13403	20521	11	9176, 2035	20533	2	2, 49	20593	5	13255, 11429
20641	7	11768, 14755	20749	2	2, 13165	20773	2	2, 5089	20809	7	9512, 12859
20857	10	19276, 2	20929	7	12710, 8773	21001	11	8192, 2038	21013	2	1, 17255
21061	7	15310, 13976	21121	19	1706, 4267	21157	2	2, 12016	21169	13	18290, 9988
21193	11	20638, 15902	21277	6	7174, 14924	21313	5	10700, 17416	21397	2	2, 4144
21433	5	5272, 1958	21481	13	1, 8999	21493	2	1, 10301	21517	5	1, 6440
21529	11	10156, 3029	21577	5	18757, 15350	21589	2	2, 3031	21601	7	4060, 20948
21613	2	1, 2357	21649	14	19786, 5327	21661	2	1, 461	21673	10	14456, 13774
21757	5	14581, 12776	21817	7	12268, 10223	21841	11	20860, 6203	21937	7	18436, 18515
21961	17	11968, 20306	21997	7	17150, 3709	22093	6	13552, 17411	22129	19	4874, 18760
22153	5	3850, 1205	22189	2	1, 986	22201	*	1, 11249	22273	5	596, 2845
22369	11	1366, 10478	22381	10	16025, 13972	22441	14	1718, 13489	22453	5	5140, 9476
22501	2	1, 20270	22549	2	1, 13214	22573	6	11153, 15028	22621	2	1, 7058
22669	2	2, 15937	22717	2	2, 11935	22741	7	16178, 16282	22777	7	15584, 12658
22801	*	6080, 5050	22861	2	2, 6448	22921	7	17522, 17704	22993	5	18682, 12599
23017	5	1, 11090	23029	2	1, 1295	23041	11	19196, 14560	23053	2	2, 18520
23173	5	1, 15995	23197	2	2, 19930	23209	31	21868, 16094	23269	6	1018, 2183
23293	5	22196, 9883	23473	5	1069, 18710	23497	5	1162, 15242	23509	2	1, 8624
23557	5	7423, 12947	23581	6	22840, 21416	23593	5	3658, 12836	23629	2	1, 1244
23677	5	9142, 4175	23689	11	5786, 2950	23761	7	10520, 11980	23773	5	1, 15002
23833	5	10393, 16862	23857	5	1072, 19442	23869	2	2707, 13469	23893	5	6061, 7046
23917	2	1, 4331	23929	7	12862, 22823	23977	5	1, 16976	24001	14	7892, 8425
24049	19	5878, 20237	24061	10	7622, 655	24097	5	3206, 20662	24109	2	2, 23602
24121	13	9304, 17294	24133	6	2344, 16832	24169	11	2710, 2675	24181	17	17026, 17621
24229	2	1, 4931	24337	5	21209, 7693	24373	7	6071, 7885	24421	7	20561, 8242
24469	14	22360, 15536	24481	11	15412, 13511	24517	5	21448, 10313	24649	*	1106, 6313
24697	5	15332, 6724	24709	2	1, 8516	24733	2	2, 21301	24781	2	1, 11570
24793	5	2044, 16271	24841	14	20692, 11495	24877	5	1, 10679	24889	11	21154, 1313
25033	5	11612, 3961	25057	5	18556, 2558	25117	5	2899, 5405	25153	10	24922, 6530
25189	2	1, 3131	25237	2	2, 23773	25261	7	9092, 12772	25309	13	1369, 19229
25321	19	19334, 17353	25357	2	1, 2330	25453	2	1, 17276	25537	10	18320, 4567
25561	11	9550, 15542	25609	7	6556, 23363	25621	10	6478, 19172	25633	5	15532, 21185
25657	5	1, 20807	25693	2	1, 25007	25717	2	1, 98	25741	6	17596, 3233
25801	7	16388, 22261	25849	7	19568, 9229	25873	10	13210, 22250	25933	2	1, 20000
25969	7	18556, 18518	25981	11	1, 14681	26017	5	15388, 1172	26029	6	15934, 5747
26041	13	6356, 13774	26053	2	1, 13421	26113	7	404, 9238	26161	13	20210, 9586
26209	11	23402, 5212	26293	6	1546, 7430	26317	6	22060, 18707	26437	5	1, 12968
26449	7	11926, 4547	26497	5	1, 13922	26557	2	1, 1226	26569	*	21484, 26120
26641	7	8024, 23239	26701	22	15712, 13418	26713	10	18358, 9077	26737	10	22966, 19802
26821	2	2, 6376	26833	5	25156, 4691	26881	11	26152, 17867	26893	5	1, 290
26953	7	21808, 1400	27061	2	1, 4259	27073	5	8692, 25346	27109	7	982, 14381
27241	17	16580, 1912	27253	2	2, 8710	27277	6	2284, 21938	27337	5	12754, 698
27361	7	21146, 17236	27397	2	1, 2891	27409	13	22234, 2438	27457	7	8776, 9440
27481	7	17672, 8440	27529	7	8462, 26809	27541	19	7064, 26839	27673	11	26026, 12506
27697	5	1, 27392	27733	2	1, 9989	27793	5	2662, 2663	27817	5	1, 16328
27889	*	1, 9080	27901	2	1, 22772	27961	13	25168, 14954	27997	5	16556, 13741

Table 8: Table of γ_1, γ_2 for RBIBD$(5q+1, 6, 1)$ construction (cont.)

q	ω	γ_1, γ_2	q	ω	γ_1, γ_2	q	ω	γ_1, γ_2	q	ω	γ_1, γ_2
28057	5	1, 19916	28069	7	7490, 10606	28081	19	2258, 5971	28201	11	11596, 27434
28297	5	1, 4175	28309	2	1, 22127	28393	15	1540, 20747	28429	2	1, 3530
28477	2	2, 24520	28513	5	22136, 16180	28537	5	27301, 2339	28549	2	1, 26345
28561	*	11900, 17422	28573	2	2, 25849	28597	2	2, 14152	28621	13	13390, 2225
28657	5	5584, 27263	28669	6	22144, 28178	28729	22	5704, 17360	28753	10	21868, 10718
28789	7	23095, 18800	28813	2	2, 3892	28837	2	1, 25505	28909	2	26773, 19544
28921	11	4856, 1852	28933	2	1, 9656	29017	5	1, 11276	29077	2	1, 5912
29101	2	2, 16819	29137	5	2180, 7261	29173	2	2, 19000	29209	7	15704, 17776
29221	2	1, 22049	29269	6	10870, 2	29389	2	1, 22643	29401	13	25006, 26942
29437	2	1, 11222	29473	5	15812, 22645	29569	17	27170, 17158	29581	10	18955, 4652
29629	7	23501, 6397	29641	7	1718, 20311	29761	17	12880, 18377	29833	5	14360, 19414
29881	7	4586, 24682	29917	2	1, 14357	29929	*	1, 22928	29989	2	2, 26377
30013	2	1, 18311	30097	10	12164, 29641	30109	2	1, 28775	30133	5	27092, 23647
30169	7	17450, 10954	30181	2	5329, 29351	30241	11	10472, 25018	30253	2	1, 10823
30313	5	21214, 12914	30469	2	1, 16979	30493	6	27863, 29215	30517	2	1, 15788
30529	13	5612, 28000	30553	5	13934, 7789	30577	5	1, 2690	30637	2	1, 14846
30649	7	11572, 6134	30661	2	1, 20540	30697	10	18512, 5626	30757	5	25480, 6938
30781	2	1, 7949	30817	5	14228, 15202	30829	2	1, 21455	30841	7	3944, 12718
30853	2	1, 3725	30937	15	20056, 1730	30949	10	27802, 9560	31033	10	1496, 27679
31069	2	1, 15086	31081	13	22708, 28232	31153	10	17116, 4904	31177	7	10100, 26794
31189	13	3484, 5732	31237	6	16054, 11132	31249	23	20, 27442	31321	7	2692, 11633
31333	5	30956, 14797	31357	2	1, 5069	31393	5	15572, 13420	31477	6	6286, 18374
31489	7	8638, 13253	31513	7	31180, 16577	31573	5	29054, 2242	31657	5	1, 18815
31729	7	21548, 16459	31741	6	30268, 16931	31849	14	30238, 26933	31873	11	5260, 15536
31957	2	1, 4562	31981	6	8270, 15340	32029	2	13717, 9266	32041	*	1, 29984
32077	2	2, 3400	32089	13	344, 7162	32173	5	11794, 18578	32233	5	21944, 4837
32257	15	15754, 9191	32341	2	2, 27484	32353	15	31364, 8821	32377	5	14768, 32086
32401	7	10090, 6638	32413	5	15232, 10499	32497	7	17752, 14936	32533	2	1, 5
32569	7	19490, 10921	32653	2	1, 26084	32713	5	18577, 8342	32749	2	1, 26279
32761	*	24934, 30617									

Table 9: Table of m, w, c, c', c'' for BIBD$(q, 13, 1)$ construction

q	m	w	c	c'	c"	q	m	w	c	c'	c"
6241	1430	23	5276	5389	3429	8737	1976	2268	2107	4412	8312
9829	6740	748	3942	3381	5212	14197	6084	5123	12586	6980	6438
15601	4157	3358	14688	10476	4395	16069	4720	14117	7813	1673	2810
16381	12288	3393	4508	6606	9926	16693	3004	5153	9988	14869	13051
18097	2895	12109	10205	1950	3646	19813	2233	7510	9356	1569	13237
20593	1341	143	1651	10214	6166	20749	6598	3119	3006	4800	9239
21061	20993	3375	12210	152	5448	21529	14249	3972	2863	19892	16577
21841	9084	8529	20542	16054	657	21997	3809	11605	16088	20775	20879
22153	8119	10588	15483	6187	16321	22621	14978	13088	20440	17145	22198
22777	14977	1740	8251	14704	20285	23557	9945	22598	21005	2174	19905
23869	13105	14848	20525	16395	21641	24181	5594	155	12600	13087	11990
24337	19291	12508	9143	22389	23346	25117	21357	274	16277	19854	9074
25741	7120	24210	24046	25039	24616	26053	22389	12522	24670	3452	9299
26209	16112	24180	21052	22252	7177	26833	6817	18598	18591	7119	21864
27457	4369	23868	296	4005	3179	28081	4962	1765	9644	21150	21612
28549	18714	7759	6616	18676	1579	29017	7565	1964	18357	4086	19246
29173	10964	9972	19507	20561	9983	29641	8581	19912	17148	29313	18919
30109	26012	20130	21915	29240	2354	30577	5687	28211	13065	20020	15930
31357	4884	10608	10496	13281	17897	31513	13169	10563	4205	25316	16700
31981	5015	12636	12148	21722	25623						

Table 10: Table of $\gamma_1, \ldots, \gamma_5$ for 13-GDD 13^q construction

q	ω	$\gamma_1, \ldots, \gamma_5$	q	ω	$\gamma_1, \ldots, \gamma_5$
13	2	2, 1, 4, 5, 9	121	*	4, 73, 98, 119, 15
157	5	20, 1, 40, 59, 117	229	6	190, 209, 152, 133, 171
277	5	218, 247, 40, 149, 207	361	*	305, 4, 116, 331, 57
397	5	61, 23, 320, 358, 297	529	*	212, 189, 376, 377, 517
601	7	223, 155, 52, 291, 2	613	2	92, 337, 184, 245, 459
661	2	634, 197, 32, 469, 495	709	2	236, 649, 118, 59, 177
733	6	602, 667, 484, 53, 549	757	2	236, 307, 496, 527, 87
829	2	496, 152, 261, 557, 583	841	*	741, 305, 452, 838, 553
853	2	155, 200, 193, 334, 393	877	2	765, 734, 167, 718, 607
937	5	497, 904, 307, 771, 836	961	*	787, 376, 543, 767, 704
997	7	586, 55, 9, 923, 944	1009	11	898, 269, 512, 561, 235
1021	10	302, 952, 1, 497, 525	1033	5	803, 393, 446, 904, 565
1069	6	401, 109, 380, 622, 489	1093	5	527, 358, 800, 783, 1009
1117	2	885, 299, 340, 1070, 1093	1129	11	856, 992, 573, 437, 709
1153	5	403, 953, 358, 3, 554	1201	11	215, 673, 416, 286, 939
1213	2	911, 760, 1100, 1179, 931	1237	2	1069, 725, 274, 1166, 669
1249	7	520, 375, 1013, 1040, 1	1297	10	971, 91, 8, 262, 783
1321	13	857, 796, 657, 20, 841	1369	*	1306, 853, 734, 443, 1035
1381	2	340, 1196, 751, 53, 585	1429	6	164, 227, 1246, 1185, 163
1453	2	947, 543, 1366, 973, 464	1489	14	1366, 1127, 1341, 31, 956
1549	2	821, 361, 1444, 927, 50	1597	11	28, 1310, 283, 1565, 1593
1609	7	1510, 1093, 1157, 513, 1358	1621	2	1167, 893, 1582, 691, 2
1657	11	460, 1370, 509, 1423, 1533	1669	2	952, 641, 1447, 609, 1586
1681	*	1244, 502, 561, 1193, 235	1693	2	61, 1409, 1305, 778, 104
1741	2	1060, 369, 134, 29, 1	1753	7	52, 781, 893, 1370, 327
1777	5	719, 1510, 867, 1154, 577	1789	6	1448, 1199, 814, 469, 651
1801	11	1037, 61, 382, 279, 1142	1849	*	1625, 525, 52, 1375, 440
1861	2	1175, 1531, 1840, 1454, 3	1873	10	4, 465, 1667, 1, 1844
1933	5	676, 1339, 1700, 11, 1329	1993	5	1819, 1858, 1385, 146, 1203
2017	5	875, 481, 297, 964, 44	2029	2	1959, 527, 1597, 1498, 2
2053	2	1660, 819, 623, 1369, 2	2089	7	275, 279, 1303, 190, 1442
2113	5	725, 1450, 806, 1485, 391	2137	10	634, 1441, 1329, 395, 1520
2161	23	1651, 837, 701, 760, 326	2197	*	1019, 1417, 1966, 470, 549
2209	*	1976, 1540, 179, 1903, 945	2221	2	784, 1484, 1031, 1881, 529
2269	2	1396, 763, 723, 731, 2	2281	7	1209, 815, 1186, 979, 38
2293	2	640, 759, 899, 1658, 1	2341	7	1966, 1199, 1465, 2126, 1215
2377	5	1714, 2084, 421, 719, 1719	2389	2	1948, 1621, 605, 1209, 2378
2401	*	2361, 1805, 1556, 1960, 1	2437	2	1180, 1519, 831, 1709, 2
2473	5	747, 745, 1942, 53, 656	2521	17	2275, 998, 389, 2284, 855
2557	2	1525, 2045, 2163, 232, 2	2593	7	657, 2494, 2587, 1391, 2048
2617	5	1883, 255, 157, 532, 1844	2677	2	1862, 352, 455, 2397, 1
2689	19	1699, 610, 305, 951, 1244	2713	5	1135, 1990, 1535, 471, 1916
2749	6	2504, 2399, 1414, 1075, 495	2797	2	2639, 1125, 1190, 1570, 1
2809	*	1157, 8, 249, 2806, 1	2833	5	1565, 2721, 1570, 206, 1
2857	11	345, 647, 883, 1234, 1598	2917	5	1300, 955, 881, 2282, 825

Table 10: Table of $\gamma_1, \ldots, \gamma_5$ for 13-GDD 13^q construction (cont.)

q	ω	$\gamma_1, \ldots, \gamma_5$	q	ω	$\gamma_1, \ldots, \gamma_5$
2953	13	73, 1382, 2920, 431, 2511	3001	14	598, 1712, 2765, 2995, 2967
3037	2	41, 2626, 411, 1472, 1	3049	11	1709, 1561, 2344, 2133, 2048
3061	6	1870, 2744, 113, 2299, 1815	3109	6	2575, 1072, 2777, 1167, 1694
3121	7	505, 2067, 1804, 1139, 1142	3169	7	425, 2439, 1555, 1624, 548
3181	7	2643, 928, 517, 71, 3062	3217	5	928, 2195, 368, 27, 1
3229	6	793, 1642, 2258, 1955, 93	3253	2	1999, 225, 2260, 2531, 2
3301	6	3027, 142, 2147, 1495, 866	3313	10	3153, 2113, 1582, 2525, 1688
3361	22	1888, 2565, 1651, 503, 1100	3373	5	1684, 2331, 689, 914, 1
3433	5	765, 1516, 1265, 1333, 2126	3457	7	2902, 95, 2839, 2660, 813
3469	2	3371, 116, 724, 939, 1	3481	*	2159, 260, 976, 759, 1
3517	2	3233, 3091, 1737, 1906, 2	3529	17	941, 2353, 1191, 2146, 1682
3541	7	3367, 430, 1190, 1223, 1035	3613	2	2217, 3160, 1937, 284, 1
3637	2	3371, 3388, 1634, 579, 1	3673	5	1101, 1409, 820, 619, 2024
3697	5	3262, 3439, 417, 1853, 1838	3709	2	3446, 1186, 593, 747, 1
3721	*	2133, 1451, 289, 1918, 1364	3733	2	121, 2981, 2122, 1923, 2
3769	7	2303, 2901, 2455, 3238, 878	3793	5	2045, 897, 409, 160, 1238
3853	2	1545, 16, 3559, 401, 2	3877	2	3755, 2404, 3098, 423, 1
3889	11	3653, 2392, 1735, 1311, 1046	4021	2	1318, 819, 2096, 3605, 1
4057	5	419, 3224, 1744, 717, 1945	4093	2	3886, 3715, 1997, 111, 2
4129	13	2236, 665, 3578, 601, 2223	4153	5	2615, 1557, 520, 3866, 1
4177	5	3892, 3525, 2288, 2183, 2545	4201	11	3723, 4168, 907, 227, 776
4261	2	2672, 4234, 1385, 669, 1	4273	5	3538, 3625, 2157, 2705, 680
4297	5	1945, 777, 796, 773, 992	4357	2	512, 1563, 2909, 1438, 1
4441	21	2807, 2311, 2061, 2740, 3356	4489	*	1901, 4126, 841, 4065, 476
4513	7	1438, 3674, 217, 1601, 2019	4549	6	2306, 2041, 520, 3083, 267
4561	11	217, 3994, 819, 173, 608	4597	5	1913, 4147, 3932, 1876, 1245
4621	2	2943, 1234, 1286, 341, 1	4657	15	3124, 3884, 911, 2479, 2715
4729	17	4438, 4141, 4295, 2288, 4359	4789	2	4035, 2026, 1862, 4349, 1
4801	7	4012, 4489, 335, 3681, 4796	4813	2	875, 4671, 1156, 1166, 1
4861	11	1864, 4124, 1907, 2383, 657	4909	6	3331, 802, 335, 4802, 3657
4933	2	3883, 1522, 3401, 1101, 2	4957	2	981, 1330, 37, 3563, 2
4969	11	2686, 4131, 4429, 131, 4172	4993	5	2823, 2770, 3953, 1736, 1
5041	*	4822, 3795, 1181, 4334, 139	5077	2	4583, 2935, 27, 2650, 2
5101	6	2450, 1, 2116, 3563, 189	5113	19	721, 3896, 928, 1565, 2973
5197	7	4589, 2065, 2800, 3752, 5175	5209	17	5165, 646, 5089, 1053, 2102
5233	10	3033, 2777, 4228, 4945, 2552	5281	7	891, 2875, 4702, 4889, 4898
5329	*	971, 3166, 1621, 4587, 1184	5413	5	1346, 1198, 1577, 4471, 4485
5437	5	148, 3341, 1129, 386, 327	5449	7	879, 187, 3676, 5387, 4034
5521	11	523, 4919, 4174, 1215, 3692	5557	2	170, 1402, 3423, 4067, 1
5569	13	2648, 59, 3562, 3675, 1	5581	6	5327, 1912, 1550, 481, 4995
5641	14	391, 4865, 1708, 5163, 2180	5653	5	350, 4090, 1535, 2479, 3117
5689	11	724, 3799, 5151, 5633, 482	5701	2	4726, 3740, 4553, 183, 1
5737	5	4876, 3344, 627, 5489, 1777	5749	2	1318, 5660, 2753, 3363, 1
5821	6	620, 1181, 718, 2071, 2685	5857	7	2485, 4163, 2806, 1923, 4034
5869	2	320, 923, 789, 1228, 1	5881	31	1213, 808, 5013, 4667, 5534
5953	7	2995, 2698, 5390, 611, 1743	6037	5	2428, 5357, 188, 277, 1485
6073	10	3363, 4559, 916, 1861, 5816	6121	7	5083, 4234, 5627, 171, 4790
6133	5	1271, 3889, 4778, 1558, 891	6217	5	2948, 4204, 5277, 4079, 1
6229	2	921, 3562, 3188, 1205, 1	6241	*	2890, 735, 3167, 1021, 320
6277	2	548, 4223, 4017, 556, 1	6301	10	1607, 3464, 4195, 2824, 1737
6337	10	5099, 1714, 465, 595, 836	6361	19	5242, 3291, 341, 4117, 4796
6373	2	944, 4, 2643, 2777, 1	6397	2	158, 5, 6081, 1936, 1
6421	6	4600, 1166, 4223, 691, 5661	6469	2	1538, 622, 2039, 1479, 1
6481	7	759, 3172, 3881, 2035, 1406	6529	7	5981, 69, 856, 5383, 5312
6553	10	4705, 2453, 5547, 1828, 392	6577	5	6317, 5991, 2995, 2950, 974
6637	2	1961, 6580, 4933, 5709, 2	6661	6	5944, 3668, 2323, 3521, 1497
6673	5	2757, 508, 1217, 536, 1	6709	2	6551, 3832, 277, 2403, 2
6733	2	878, 3573, 3077, 6010, 1	6781	2	5584, 4736, 3749, 477, 1
6793	10	2566, 1, 3389, 2613, 4856	6829	2	2517, 650, 4949, 5848, 1
6841	22	3197, 139, 5128, 4803, 5756	6889	*	2476, 2459, 5625, 1268, 1
6949	2	5757, 3866, 1708, 4661, 1	6961	13	1921, 508, 4733, 2421, 6320
6997	5	5111, 1324, 986, 1111, 855	7057	5	2266, 2351, 5234, 3627, 1
7069	2	1216, 4238, 6743, 6327, 6493	7129	7	5267, 2176, 3903, 1729, 878
7177	10	4397, 610, 5163, 6841, 1964	7213	5	5528, 6597, 5531, 7006, 1
7237	2	1070, 4276, 1463, 5817, 1	7297	5	1675, 4833, 893, 5980, 2054
7309	6	4473, 2506, 1295, 253, 2402	7321	7	99, 5555, 157, 3574, 3158
7333	6	6277, 4138, 4376, 521, 4137	7369	7	6353, 2767, 5865, 142, 1316
7393	5	4197, 2141, 1150, 7111, 5768	7417	5	5439, 1678, 7039, 209, 1490
7477	2	2950, 2906, 3345, 4985, 1	7489	7	2217, 568, 5351, 5449, 1520
7537	7	1574, 697, 2843, 1768, 6267	7549	2	1155, 1786, 1208, 1097, 1
7561	13	4133, 5050, 3423, 4837, 458	7573	2	1163, 3585, 1687, 1582, 2
7621	2	1949, 5398, 7496, 4797, 1	7669	2	201, 4595, 1468, 5497, 2
7681	17	3145, 2602, 483, 35, 7448	7717	2	952, 2283, 2371, 2249, 2
7741	7	640, 56, 2335, 1931, 3243	7753	10	1498, 6669, 857, 3764, 1921
7789	2	7508, 4725, 5147, 5314, 1	7873	5	4190, 1522, 3947, 7137, 1
7921	*	2818, 5487, 2900, 6245, 1	7933	2	7157, 4688, 1527, 1594, 1
7993	5	1037, 4609, 4413, 3514, 6080	8017	5	3929, 6883, 6268, 5259, 7496
8053	2	4083, 3923, 3094, 4699, 2	8089	17	6832, 641, 6805, 2451, 4958
8101	6	5864, 3970, 2113, 989, 4131	8161	7	6940, 1238, 7331, 6435, 1
8209	7	7601, 1, 411, 7948, 4124	8221	2	844, 1365, 4883, 4567, 2
8233	10	2086, 7133, 6763, 3567, 6032	8269	2	1013, 3830, 2098, 7623, 1
8293	2	8051, 3, 6770, 1426, 1	8317	6	5308, 1, 4376, 3227, 7911
8329	7	7973, 1522, 273, 6289, 2894	8353	5	4653, 4078, 5024, 4307, 1
8377	5	171, 5057, 7150, 7250, 199	8389	6	4739, 1, 5434, 2612, 5235
8461	6	7957, 7955, 2049, 4990, 1490	8521	13	5831, 5323, 3201, 3166, 5282
8581	6	3208, 1, 7751, 7569, 7358	8629	6	1493, 3248, 1510, 2329, 3483
8641	17	2323, 6071, 7204, 6219, 2588	8677	2	4865, 598, 5421, 4273, 2
8689	13	2547, 1129, 394, 7991, 7484	8713	5	7883, 5132, 6970, 2163, 1
8737	5	8620, 1760, 4097, 3693, 1	8761	23	4215, 3484, 3919, 8735, 7568
8821	2	2853, 2558, 1552, 7193, 1	8893	5	3374, 5637, 2284, 2627, 1
8929	11	1137, 4030, 5549, 7136, 1	8941	6	3616, 1733, 5888, 1633, 8007
9001	7	4805, 6578, 4696, 2397, 1	9013	5	1997, 5744, 6931, 7024, 2007
9049	7	2632, 8579, 2875, 6219, 476	9109	10	2881, 1294, 8126, 7865, 6165
9133	6	3463, 7312, 3665, 4268, 1821	9157	6	7774, 3039, 2551, 1367, 3038
9181	2	5593, 6562, 5247, 215, 2	9241	13	3199, 3448, 6035, 3801, 1724
9277	5	2927, 5668, 5318, 213, 1	9337	5	4231, 8061, 2638, 1379, 3788
9349	2	3465, 2, 4108, 6617, 1	9397	2	722, 5645, 1780, 8481, 1
9409	*	5535, 3701, 2044, 2275, 3332	9421	2	2479, 2542, 5015, 783, 2
9433	5	4185, 1930, 2329, 1865, 9386	9601	13	4179, 8104, 8735, 7339, 4052
9613	2	262, 765, 6659, 9601, 2	9649	7	4675, 7546, 909, 5915, 5240
9661	2	8187, 3110, 8069, 9430, 1	9697	10	3139, 6771, 8663, 3316, 5816
9721	7	3465, 563, 6880, 6110, 1	9733	2	6674, 5, 8038, 2763, 1
9769	13	2777, 5194, 3571, 9483, 3200	9781	6	1718, 5926, 4673, 2059, 8475
9817	5	9469, 2, 9148, 4577, 4443	9829	10	4370, 4546, 6687, 5969, 8407
9901	2	1834, 9133, 1059, 9683, 2	9949	2	263, 1419, 3937, 9178, 2
9973	11	6520, 4903, 9071, 3662, 3093	10009	11	6817, 6207, 8824, 2273, 7316
10069	2	2056, 2067, 6362, 6713, 1	10093	2	6190, 6584, 7863, 2381, 1
10141	2	3969, 5740, 1232, 7697, 1	10177	7	9400, 7124, 4405, 2315, 3993
10201	*	9405, 3899, 1160, 9808, 1	10273	10	8907, 346, 2515, 5711, 7220
10321	7	1981, 1480, 1239, 3815, 7754	10333	5	2227, 3284, 10294, 1853, 3069
10357	2	1279, 4923, 2231, 7870, 2	10369	13	5961, 2716, 995, 6163, 6818
10429	7	2225, 3010, 6566, 1411, 7095	10453	5	10052, 9964, 7733, 303, 325

Table 10: Table of $\gamma_1, \ldots, \gamma_5$ for 13-GDD 13^q construction (cont.)

q	ω	$\gamma_1, \ldots, \gamma_5$	q	ω	$\gamma_1, \ldots, \gamma_5$
10477	2	5572, 2347, 2703, 6329, 2	10501	2	5092, 7214, 1889, 3045, 1
10513	7	667, 10300, 8949, 17, 6284	10597	5	7725, 2794, 5741, 4232, 1
10609	*	7241, 208, 3735, 9541, 9152	10657	7	3892, 1, 716, 9857, 9501
10729	7	2597, 5218, 1735, 8523, 10004	10753	11	4370, 7025, 8671, 9082, 5499
10789	2	3621, 9370, 3764, 5813, 1	10837	2	10499, 10256, 8416, 9819, 1
10861	2	7874, 3802, 6413, 2685, 1	10909	2	6975, 6086, 953, 4708, 1
10957	5	5657, 4732, 8073, 4682, 1	10993	7	5767, 10335, 9046, 5633, 8420
11113	13	1673, 10982, 6982, 3793, 393	11149	10	9781, 3316, 10973, 8450, 6813
11161	7	10901, 550, 4779, 4676, 1	11173	5	9737, 1629, 7558, 4730, 1
11197	2	3469, 10721, 7149, 5374, 2	11257	10	9147, 2776, 4067, 7147, 3272
11317	2	203, 10268, 9994, 1479, 1	11329	7	213, 6988, 7721, 10508, 1
11353	7	8629, 10726, 10593, 7595, 2834	11437	2	320, 2109, 3965, 6706, 1
11449	*	6412, 5726, 6639, 3527, 1	11497	7	7287, 11123, 8938, 49, 830
11593	5	11291, 11320, 11233, 7743, 9860	11617	10	5275, 5416, 10059, 9725, 6992
11677	2	7840, 4501, 3135, 10331, 2	11689	7	4859, 6154, 6565, 1671, 9446
11701	6	5817, 4330, 9055, 6545, 9536	11821	2	4139, 3, 1226, 8260, 2023
11833	5	7078, 10259, 11361, 9961, 8684	11881	*	10097, 1210, 4101, 9145, 8360
11941	10	2782, 2273, 10995, 11906, 5575	11953	5	8199, 11494, 3653, 1862, 1
12037	5	10313, 5084, 5901, 4726, 3043	12049	13	10717, 10048, 2733, 2099, 9290
12073	7	7642, 8768, 3403, 875, 3219	12097	5	4317, 50, 3581, 3406, 1
12109	6	11224, 1, 6845, 10160, 10953	12157	2	10697, 11924, 6730, 2199, 1
12241	7	8695, 4774, 131, 2259, 1994	12253	2	10239, 4888, 545, 4952, 1
12277	2	2477, 7462, 2420, 807, 1	12289	11	4579, 3472, 3231, 1529, 4172
12301	2	4167, 5932, 767, 6062, 1	12373	2	8433, 962, 12239, 6826, 1
12409	7	10269, 2488, 383, 7537, 7448	12421	7	9223, 3604, 7697, 1388, 10581
12433	13	10913, 1, 8362, 2697, 6362	12457	10	8753, 3400, 6709, 9, 6800
12517	6	11585, 2404, 4442, 10651, 10113	12541	14	4712, 6400, 11253, 863, 4735
12553	5	4555, 7852, 8763, 8177, 9350	12577	10	7849, 513, 340, 9017, 4880
12589	2	9069, 38, 11554, 12437, 1	12601	11	9855, 8836, 11513, 4081, 10718
12613	2	4162, 3, 8078, 10811, 1	12637	2	11750, 9256, 5535, 1871, 1
12697	7	4513, 8254, 12689, 8445, 2468	12721	13	11773, 6280, 7433, 9129, 1934
12757	2	1991, 4261, 10096, 8457, 2	12769	*	5753, 2631, 1798, 4526, 1
12781	2	6881, 3579, 1228, 1706, 1	12829	2	6239, 6944, 8733, 922, 1
12841	21	10096, 1, 11957, 7737, 9590	12853	5	9506, 11871, 9844, 6965, 1
12889	13	11357, 9982, 7963, 10251, 5864	12973	14	10349, 10996, 3031, 12866, 237
13009	7	9322, 12525, 617, 9343, 2582	13033	5	8825, 10012, 1927, 10983, 5006
13093	6	7240, 1, 3209, 11480, 5931	13177	5	8962, 8720, 603, 6737, 1
13249	7	4685, 11025, 7900, 7243, 11582	13297	5	2360, 11794, 6155, 4023, 12055
13309	6	10667, 640, 10381, 5288, 639	13381	10	658, 2330, 7595, 3849, 7855
13417	5	3769, 12574, 3533, 13101, 10640	13441	11	6493, 8032, 8651, 13383, 13328
13477	2	6733, 2578, 5673, 8891, 2	13513	5	863, 1240, 2612, 13161, 1
13537	7	6519, 5530, 7385, 13435, 9542	13597	5	8062, 2756, 7693, 8909, 723
13633	5	3893, 4660, 10101, 12098, 1	13669	6	8775, 4307, 10276, 6068, 1
13681	22	12161, 12352, 3097, 8378, 2769	13693	6	215, 6652, 829, 1238, 7041
13729	23	11331, 11812, 3691, 125, 4220	13789	7	6820, 489, 10753, 12431, 10700
13873	5	2207, 3838, 6159, 4279, 4076	13921	7	12993, 316, 12301, 13397, 632
13933	2	11753, 2, 5278, 5289, 1	14029	6	8572, 1532, 6635, 7717, 13263
14149	6	4133, 13268, 7540, 3019, 2205	14173	2	11009, 6339, 2068, 2587, 2
14197	11	2474, 6664, 9283, 12821, 8331	14221	2	7486, 3, 13994, 203, 7951
14281	19	1381, 7468, 9777, 725, 12674	14293	6	9688, 2936, 3287, 13261, 12825
14341	2	12058, 2411, 5929, 9285, 2	14389	2	5565, 3196, 11522, 2987, 1
14401	11	2269, 10264, 12207, 4373, 2966	14437	5	11198, 8989, 13529, 3268, 1689
14449	22	3793, 7912, 11661, 2945, 3956	14461	2	12538, 12488, 3605, 5193, 1
14533	2	227, 4123, 10305, 4036, 2	14557	2	8693, 8662, 949, 2541, 2
14593	5	8919, 4184, 832, 3431, 1	14629	2	12635, 9776, 11890, 13401, 13699
14641	*	13628, 8781, 12760, 2183, 1	14653	2	7517, 3092, 5182, 10527, 1
14713	5	14411, 13996, 759, 2923, 13280	14737	10	14483, 1588, 9177, 1291, 3176
14797	2	3988, 11867, 12038, 12813, 1	14821	2	9781, 7677, 9778, 8819, 2
14869	2	983, 1838, 4479, 10984, 1	14929	7	3860, 12568, 2105, 6633, 1
15013	2	6383, 4, 11762, 13167, 1	15061	2	5417, 12561, 14528, 11380, 1
15073	5	10107, 4097, 4504, 11870, 1	15121	11	10325, 952, 3471, 6211, 9380
15193	5	3275, 3362, 5055, 13234, 1	15217	10	14725, 12508, 15083, 13305, 3668
15241	11	13799, 7000, 8307, 3265, 11120	15277	6	2569, 14104, 8285, 12674, 1173
15289	11	8769, 2422, 5189, 12217, 338	15313	5	7054, 11456, 12633, 1769, 1
15349	2	7941, 12485, 1054, 1142, 1	15361	7	11158, 9029, 9025, 11631, 2696
15373	2	165, 851, 9062, 3406, 1	15493	5	9698, 682, 2173, 2567, 3033
15541	6	12461, 8080, 9865, 13107, 11810	15601	23	1142, 5374, 4081, 1109, 5055
15625	*	3820, 11714, 6323, 7011, 1	15649	11	8515, 14746, 14999, 1335, 2720
15661	2	3713, 6602, 5355, 1942, 1	15733	6	1526, 4348, 10855, 7241, 2955
15817	5	125, 1052, 5752, 6261, 1	15877	5	5047, 10826, 5920, 14615, 3975
15889	21	5039, 3550, 8229, 12499, 1046	15901	10	7481, 7934, 15880, 11017, 9111
15913	5	11459, 10305, 8302, 6050, 10399	15937	7	9250, 11168, 10333, 9701, 10641
15973	7	4856, 967, 826, 2945, 5979	16033	5	7495, 11842, 8957, 11703, 7652
16057	7	1601, 8284, 12109, 614, 663	16069	2	8763, 2, 658, 7853, 1
16129	*	875, 4864, 16089, 13435, 9344	16141	6	10300, 8495, 15501, 6697, 7646
16189	2	8405, 6140, 465, 15772, 1	16249	17	14037, 1552, 16165, 10781, 3452
16273	7	15827, 295, 3592, 933, 4628	16333	2	7953, 14950, 12290, 3509, 1
16369	7	1409, 1, 1534, 1053, 2384	16381	2	1078, 10208, 15975, 6767, 1
16417	10	14783, 11281, 16191, 6928, 12104	16453	2	1856, 8164, 6993, 14711, 1
16477	2	11056, 7153, 2219, 10755, 2	16561	7	15981, 14656, 14819, 5479, 16112
16573	2	8201, 4693, 2703, 1870, 2	16633	15	14043, 5080, 1337, 3529, 1640
16657	5	938, 3088, 213, 12755, 9163	16693	2	903, 4, 4823, 12667, 2
16729	13	15853, 7671, 13667, 5098, 4898	16741	6	16262, 8080, 2537, 5755, 8661
16921	17	903, 94, 3907, 983, 12920	16981	2	12410, 7629, 10691, 2914, 1
16993	10	8063, 1474, 6105, 11467, 2948	17029	10	4799, 11056, 4064, 2845, 14847
17041	7	4750, 1, 16395, 13481, 11060	17053	2	3249, 7177, 11771, 4270, 2
17077	2	5452, 6038, 12981, 14411, 1	17137	5	16363, 9904, 15381, 2165, 16760
17161	*	4906, 14552, 12299, 9219, 1	17209	14	12434, 12448, 15383, 6031, 11241
17257	5	4299, 6658, 5209, 953, 7484	17293	7	7525, 13666, 12992, 15287, 9489
17317	2	3069, 12844, 14375, 277, 2	17341	6	7412, 1, 1756, 3125, 5409
17377	7	9255, 8428, 14375, 9469, 16856	17389	2	1773, 2, 11843, 10816, 1
17401	11	1067, 1, 11745, 2446, 2228	17449	14	1947, 12856, 8755, 161, 11186
17497	5	2977, 7811, 4252, 8679, 13184	17509	2	11475, 10085, 15400, 8480, 1
17569	11	547, 4708, 2259, 13199, 17564	17581	10	4835, 7910, 2661, 15742, 3955
17713	7	1135, 3784, 644, 6551, 7695	17737	7	4963, 4792, 9893, 7617, 9584
17749	2	10930, 4799, 2727, 8977, 2	17761	19	15973, 13648, 2741, 10383, 17426
17881	7	775, 2056, 11807, 10059, 10520	17929	11	2279, 5380, 17263, 1335, 16508
17977	5	7107, 5576, 2476, 6383, 1	17989	2	5878, 2120, 15059, 16155, 1
18013	2	976, 7430, 8669, 3, 1	18049	13	8999, 13900, 7587, 7273, 10544
18061	6	6355, 2146, 16703, 15686, 15915	18097	5	3813, 10132, 13933, 17651, 14114
18121	23	6781, 4336, 14120, 1337, 3633	18133	5	874, 10671, 8465, 2948, 1
18169	11	16143, 6280, 7169, 14239, 3140	18181	2	16186, 14168, 10931, 6819, 1
18217	7	721, 14410, 7593, 8369, 10604	18229	2	16997, 8533, 5728, 12675, 2
18253	5	3476, 14860, 3359, 5083, 10185	18289	13	2356, 16841, 15853, 13479, 14810
18301	6	7141, 1462, 14591, 10166, 16839	18313	10	4961, 17488, 17835, 3763, 15884
18397	6	12139, 3568, 16838, 3503, 14829	18433	5	13647, 6844, 9065, 11245, 6296
18457	5	6388, 4787, 11953, 3279, 9572	18481	13	8711, 8704, 11371, 10383, 3824
18493	2	13217, 3241, 9441, 17614, 2	18517	6	17864, 16918, 17189, 1909, 1599
18541	6	11845, 10774, 2249, 5228, 7767	18553	5	14653, 10852, 10793, 1425, 15734
18637	2	14335, 9087, 18418, 17093, 2	18661	10	3557, 2140, 2510, 9661, 6777
18757	2	2783, 1792, 10100, 6633, 1	18769	*	2691, 9521, 18226, 6482, 1
18793	5	6369, 358, 14327, 92, 1	18913	7	7823, 9352, 1963, 14157, 9290
18973	2	9700, 380, 14237, 2937, 1	19009	23	2763, 11044, 11191, 5981, 1190

Table 10: Table of $\gamma_1, \ldots, \gamma_5$ for 13-GDD 13^q construction (cont.)

q	ω	$\gamma_1, \ldots, \gamma_5$	q	ω	$\gamma_1, \ldots, \gamma_5$
19069	2	13649, 13610, 2746, 17367, 1	19081	17	17378, 10906, 4331, 18937, 10737
19141	2	10484, 1913, 4612, 18117, 1	19213	5	3860, 1, 12340, 1841, 10821
19237	2	6009, 17590, 2858, 2675, 1	19249	7	12475, 5728, 17927, 16701, 5222
19273	5	5090, 2404, 18543, 13925, 18127	19309	6	13244, 12454, 3859, 7475, 15423
19321	*	8091, 5740, 6007, 17573, 140	19333	2	15957, 13490, 12514, 13547, 1
19381	7	19346, 5315, 19054, 12451, 5463	19417	5	16322, 4281, 10588, 11411, 1
19429	6	18817, 11350, 3272, 15335, 8079	19441	13	12489, 1, 784, 15377, 11894
19477	6	14753, 12616, 3817, 16850, 6861	19489	19	12171, 7666, 6565, 14957, 14276
19501	2	9488, 10853, 3508, 18699, 1	19597	2	12695, 3586, 6776, 16701, 1
19609	13	11593, 13265, 6591, 8716, 3680	19681	11	10547, 7420, 2911, 17169, 17444
19717	2	1961, 2, 5871, 2266, 1	19753	5	16875, 11984, 5483, 12040, 1
19777	11	15206, 1, 5944, 17477, 8325	19801	13	9463, 19775, 15298, 1509, 15560
19813	2	5219, 67, 15555, 8620, 2	19861	11	5533, 8045, 3548, 13768, 5073
19993	10	11195, 9028, 6367, 13971, 18056	20029	2	19931, 18508, 12998, 1059, 1
20089	7	10737, 17386, 18475, 9197, 9188	20101	6	14444, 3382, 14987, 4069, 16719
20113	10	11595, 11512, 5231, 14893, 2912	20149	2	10792, 17043, 8918, 19313, 2209
20161	13	16571, 18412, 8161, 16713, 19286	20173	2	6226, 8593, 5195, 19797, 2
20233	5	6205, 2133, 7331, 5794, 956	20269	2	6671, 6487, 14559, 6586, 2
20341	2	18440, 13773, 9533, 19468, 1	20353	5	9981, 18646, 17479, 14681, 15032
20389	6	10556, 13589, 7912, 2197, 14961	20509	2	16353, 2, 2272, 16691, 1
20521	11	7971, 1582, 7721, 9655, 9176	20533	2	8788, 835, 15423, 3677, 2
20593	5	18965, 8690, 19372, 19797, 13255	20641	7	1533, 1, 3977, 10546, 11768
20749	2	15429, 4, 12659, 14161, 2	20773	2	5219, 17143, 4719, 14920, 2
20809	7	895, 8242, 339, 7949, 9512	20857	10	11091, 19276, 10901, 1561, 17696
20929	7	14727, 17182, 17905, 683, 12710	21001	11	1945, 15670, 13751, 5631, 8192
21013	2	19089, 19526, 20782, 8021, 1	21061	7	16316, 15310, 13991, 9589, 18045
21121	19	19571, 3412, 6211, 11907, 1706	21157	2	15061, 6135, 11818, 2837, 2
21169	13	11236, 1, 6791, 16365, 18290	21193	11	17785, 20638, 9107, 392, 15081
21277	6	1490, 7174, 1577, 8503, 14103	21313	5	13545, 11299, 19138, 14795, 10700
21397	2	21159, 985, 1847, 15532, 2	21433	5	21021, 10544, 11110, 5759, 1
21481	13	9911, 9742, 7923, 17846, 1	21493	2	15766, 5720, 10683, 4355, 1
21517	5	9233, 5829, 12112, 10406, 1	21529	11	12809, 10156, 9633, 10915, 1886
21577	5	298, 19790, 19593, 8915, 18757	21589	2	829, 21255, 7756, 15785, 2
21601	7	19205, 4060, 8383, 8553, 8120	21613	2	8770, 15104, 10883, 11235, 1
21649	14	13345, 19786, 15147, 2591, 17924	21661	2	13289, 2, 13576, 6999, 1
21673	10	3987, 19114, 809, 21313, 14456	21757	5	12089, 5368, 20198, 3057, 14581
21817	7	557, 20132, 7105, 17578, 16629	21841	11	18833, 20860, 8331, 5809, 19880
21937	7	17663, 14936, 907, 16342, 8325	21961	17	6569, 11968, 10807, 7827, 18182
21997	7	13232, 3982, 3499, 6551, 11559	22093	6	7373, 13552, 6302, 15181, 8541
22129	19	7777, 18921, 6004, 2627, 4874	22153	5	4061, 3850, 12549, 7093, 15458
22189	2	21467, 7724, 1672, 8487, 1	22201	*	14560, 11249, 20114, 20571, 1
22273	5	7805, 10099, 4138, 22029, 596	22369	11	5089, 1366, 18023, 1239, 17672
22381	10	5569, 11956, 11063, 873, 6356	22441	14	10687, 18459, 4907, 7204, 1718
22453	5	18431, 5140, 21008, 22225, 4383	22501	2	8529, 13462, 3722, 7727, 1
22549	2	4331, 2, 11266, 14253, 1	22573	6	22438, 11153, 13563, 6103, 11420
22621	2	22301, 4522, 5852, 16743, 1	22669	2	20743, 11458, 11573, 1875, 2
22717	2	16601, 1621, 3549, 5374, 2	22741	7	8266, 16178, 3989, 943, 22137
22777	7	19168, 1393, 15017, 12014, 6021	22801	*	4099, 11704, 3467, 5409, 6080
22861	2	3814, 21869, 4903, 747, 2	22921	7	2673, 18076, 8381, 7267, 17522
22993	5	2377, 18682, 8511, 12539, 17906	23017	5	5391, 21578, 16444, 6623, 1
23029	2	8474, 243, 19613, 16468, 1	23041	11	15395, 9202, 19899, 703, 19196
23053	2	17314, 15769, 5189, 7671, 2	23173	5	8495, 14871, 20900, 22918, 1
23197	2	7847, 489, 21439, 10162, 2	23209	31	1400, 21868, 19367, 7681, 4629
23269	6	22451, 1018, 21259, 7682, 22251	23293	5	7549, 22196, 19234, 3035, 5295
23473	5	8302, 8873, 14384, 10071, 1069	23497	5	153, 1162, 16109, 11719, 5216
23509	2	5279, 3712, 18074, 4857, 1	23557	5	4846, 1220, 19029, 16667, 7423
23581	6	21805, 22840, 13424, 1727, 741	23593	5	14337, 3658, 719, 22723, 7316
23629	2	20212, 19670, 737, 7089, 1	23677	5	4415, 9142, 12349, 7310, 4185
23689	11	22135, 11572, 18041, 18513, 5786	23761	7	917, 17620, 18847, 9939, 10520
23773	5	23465, 21261, 23738, 17752, 1	23833	5	5848, 2325, 3155, 5042, 10393
23857	5	15519, 18464, 22048, 11903, 1	23869	2	16528, 7595, 17834, 15105, 2707
23893	5	23223, 19268, 11554, 5735, 6061	23917	2	20105, 2, 17434, 11727, 1
23929	7	23177, 12862, 4137, 9913, 7520	23977	5	22695, 4822, 1007, 10226, 1
24001	14	10693, 7492, 2913, 12335, 7892	24049	19	10033, 5878, 2757, 5993, 5984
24061	10	2239, 7622, 2944, 12593, 2283	24097	5	1489, 23188, 14493, 11825, 3206
24109	2	15250, 19773, 8923, 19367, 2	24121	13	9292, 19287, 4825, 9347, 6038
24133	6	9854, 2344, 15437, 3601, 21789	24169	11	17881, 2710, 10889, 9218, 19485
24181	17	4159, 17026, 17135, 18608, 5319	24229	2	154, 977, 19070, 4599, 1
24337	5	20690, 21209, 8493, 16090, 12877	24373	7	15205, 6071, 6506, 23170, 23643
24421	7	10826, 20561, 5932, 14617, 22041	24469	14	9991, 22360, 8366, 155, 4557
24481	11	2843, 9748, 20497, 10821, 15842	24517	5	7081, 21448, 10655, 8882, 22425
24649	*	10957, 2212, 14399, 10251, 1106	24697	5	2223, 7667, 727, 21388, 15332
24709	2	2445, 2, 526, 19565, 1	24733	2	1594, 12193, 2085, 12785, 2
24781	2	22864, 3, 10940, 4415, 1	24793	5	16743, 4088, 155, 4936, 1
24841	14	1671, 20692, 3499, 15983, 16544	24877	5	2182, 10347, 9761, 7790, 1
24889	11	7233, 10822, 23467, 11567, 11552	25033	5	11457, 24346, 23191, 24659, 11612
25057	5	829, 18556, 5487, 8735, 12056	25117	5	1145, 21746, 8686, 3801, 2899
25153	10	145, 24922, 23259, 23987, 15062	25189	2	9609, 23740, 18320, 10265, 1
25237	2	14853, 2848, 22229, 14779, 2	25261	7	21826, 9092, 4553, 14551, 16671
25309	13	21256, 20054, 12767, 11235, 1369	25321	19	1169, 14548, 15061, 7245, 19334
25357	2	23795, 18664, 20474, 4911, 1	25453	2	9491, 24296, 11914, 2349, 1
25537	10	17443, 11104, 4061, 10071, 18320	25561	11	7351, 5992, 6975, 12671, 2996
25609	7	13495, 15892, 8255, 19041, 20750	25621	10	14599, 6478, 5435, 23924, 4815
25633	5	7259, 15532, 19773, 5605, 13544	25657	5	11699, 21400, 6524, 5781, 1
25693	2	4173, 23828, 18058, 7319, 1	25717	2	6893, 4154, 9297, 8446, 1
25741	6	23192, 17596, 8551, 11111, 8145	25801	7	22601, 1, 10264, 2661, 16388
25849	7	12343, 25660, 3303, 21233, 19568	25873	10	5439, 13210, 24911, 18751, 548
25933	2	20561, 2, 16036, 12867, 1	25969	7	25829, 22348, 16993, 1935, 24158
25981	11	5972, 1, 16408, 11861, 1203	26017	5	15062, 5499, 20975, 12244, 1
26029	6	25939, 15934, 1832, 13961, 10095	26041	13	25093, 21586, 21029, 23367, 6356
26053	2	23097, 4256, 11903, 13126, 1	26113	7	6791, 1, 9016, 18507, 404
26161	13	8559, 1, 24298, 3785, 20210	26209	11	9081, 25360, 21437, 20815, 23402
26293	6	13061, 1546, 5869, 13670, 24747	26317	6	22331, 22060, 5078, 15565, 4257
26437	5	16922, 13923, 4853, 14116, 1	26449	7	25171, 11926, 23687, 24861, 2024
26497	5	8991, 16004, 8518, 16523, 1	26557	2	1293, 7102, 18476, 16001, 1
26569	*	4245, 9184, 12937, 22949, 17876	26641	7	15245, 1, 16047, 6886, 8024
26701	22	3361, 15712, 16814, 4733, 14385	26713	10	12861, 18358, 18031, 21017, 11756
26737	10	19453, 22966, 8573, 3081, 19196	26821	2	21575, 25639, 20488, 16671, 2
26833	5	5338, 5486, 19187, 9531, 1	26881	11	9763, 17164, 20651, 10239, 12548
26893	5	25828, 1881, 671, 23360, 1	26953	7	14146, 5420, 2917, 15251, 6117
27061	2	2829, 2, 16396, 8465, 1	27073	5	18143, 8692, 17277, 16285, 11402
27109	7	18932, 982, 11243, 9589, 8577	27241	17	8681, 16186, 55, 22329, 16580
27253	2	17019, 22813, 665, 6724, 2	27277	6	6623, 2284, 15829, 19112, 24993
27337	5	22195, 12754, 23943, 20915, 25508	27361	7	12112, 1, 6167, 19299, 21146
27397	2	700, 10883, 23108, 27051, 1	27409	13	5555, 22234, 9945, 8101, 15722
27457	7	741, 12082, 668, 7277, 8671	27481	7	12287, 1390, 27445, 13287, 17672
27529	7	1457, 17620, 7237, 5253, 8462	27541	19	5741, 10959, 22603, 11308, 7064
27673	11	24494, 26026, 23657, 727, 9219	27697	5	10670, 9208, 12615, 10295, 1
27733	2	17259, 18860, 8110, 425, 1	27793	5	25881, 2662, 5483, 15817, 9224
27817	5	19604, 9784, 24065, 8289, 1	27889	*	14234, 27508, 83, 1209, 1
27901	2	17453, 2, 21130, 11511, 1	27961	13	22989, 25168, 8699, 17654, 21625
27997	5	27016, 16556, 14647, 14225, 4731	28057	5	1547, 16246, 13736, 17739, 1
28069	7	24575, 7209, 22471, 19060, 7490	28081	19	4749, 21202, 7891, 7097, 2258

Table 10: Table of $\gamma_1, \ldots, \gamma_5$ for 13-GDD 13^q construction (cont.)

q	ω	$\gamma_1, \ldots, \gamma_5$	q	ω	$\gamma_1, \ldots, \gamma_5$
28201	11	13367, 11596, 23349, 4357, 5342	28297	5	28211, 140, 2715, 13030, 1
28309	2	16095, 20812, 21086, 623, 1	28393	15	1262, 1540, 16699, 22511, 1509
28429	2	7839, 2, 27724, 10529, 1	28477	2	7227, 4, 21715, 24917, 2
28513	5	9717, 21700, 5665, 1889, 22136	28537	5	6430, 27338, 24225, 21203, 27301
28549	2	3335, 2, 892, 21501, 1	28561	*	22209, 8311, 8512, 13451, 11900
28573	2	11817, 28297, 13349, 28258, 2	28597	2	26326, 14131, 5481, 17825, 2
28621	13	14416, 17455, 2303, 22214, 4065	28657	5	8913, 5584, 13789, 5741, 27932
28669	6	4087, 22144, 15620, 845, 6525	28729	22	9007, 23044, 485, 7341, 11522
28753	10	12919, 21868, 12969, 14477, 24272	28789	7	4907, 23095, 17014, 19190, 16149
28813	2	12927, 13825, 24748, 7733, 2	28837	2	11195, 8854, 9452, 24783, 1
28909	2	2573, 3, 27338, 11728, 26773	28921	11	19989, 1, 23266, 20291, 4856
28933	2	24923, 26564, 4918, 12813, 1	29017	5	7617, 6880, 28430, 22415, 1
29077	2	19209, 11452, 6938, 24941, 1	29101	2	6430, 9671, 27337, 7119, 2
29137	5	21095, 4360, 15079, 27525, 2180	29173	2	21439, 22409, 26469, 15046, 2
29209	7	17615, 21484, 12177, 29047, 15704	29221	2	27195, 2, 23926, 857, 1
29269	6	20483, 9472, 12409, 20396, 18399	29389	2	2871, 2, 8668, 28229, 1
29401	13	25797, 25006, 1729, 29375, 27890	29437	2	12663, 9188, 15724, 22751, 1
29473	5	23759, 16383, 19246, 3793, 15812	29569	17	6777, 1, 3820, 28271, 27170
29581	10	10589, 8330, 22359, 8014, 18955	29629	7	10160, 16501, 190, 8663, 29385
29641	7	7449, 3436, 9199, 6869, 1718	29761	17	1349, 12880, 8629, 22449, 22244
29833	5	24821, 18964, 9687, 11215, 14360	29881	7	9751, 5170, 8475, 12425, 4586
29917	2	22791, 25840, 28208, 29255, 1	29929	*	25288, 15137, 11079, 17276, 1
29989	2	13541, 6793, 10072, 5289, 2	30013	2	10547, 3, 15572, 22498, 1
30097	10	11347, 10612, 12525, 965, 12164	30109	2	14104, 7946, 19805, 24459, 1
30133	5	16330, 27092, 21113, 9553, 15627	30169	7	4853, 4732, 6615, 11209, 17450
30181	2	21392, 3646, 22509, 28229, 5329	30241	11	1901, 20944, 8965, 18201, 10472
30253	2	9047, 5410, 10832, 16395, 1	30313	5	20819, 21214, 23061, 541, 10682
30469	2	23643, 7562, 19493, 24880, 1	30493	6	1192, 27863, 17349, 21541, 2630
30517	2	3689, 24430, 20612, 15633, 1	30529	13	6719, 11224, 9301, 30501, 5612
30553	5	9611, 27868, 10927, 4047, 13934	30577	5	20411, 4558, 17540, 18195, 1
30637	2	30603, 11954, 1186, 21761, 1	30649	7	13197, 11572, 7817, 29131, 25778
30661	2	4270, 17066, 9209, 29499, 1	30697	10	7021, 19762, 7797, 3047, 18512
30757	5	5993, 25480, 2480, 27763, 24123	30781	2	6045, 15046, 16910, 10109, 1
30817	5	6963, 28456, 10261, 21785, 14228	30829	2	22779, 4270, 7466, 8507, 1
30841	7	13439, 14338, 4687, 7815, 3944	30853	2	30791, 3490, 5594, 6843, 1
30937	15	12313, 20056, 4562, 3353, 29109	30949	10	30680, 27802, 18107, 3361, 9441
31033	10	3323, 20944, 2643, 30193, 1496	31069	2	6983, 2, 18261, 14350, 1
31081	13	4443, 22708, 13103, 21883, 15518	31153	10	24185, 17116, 24309, 19087, 3080
31177	7	21988, 10100, 13697, 13873, 15603	31189	13	22541, 3484, 3434, 13297, 20583
31237	6	28064, 16054, 17845, 18425, 15183	31249	23	26105, 3933, 28906, 26863, 20
31321	7	3167, 24628, 28611, 12949, 12314	31333	5	23639, 30956, 12220, 19825, 11637
31357	2	15443, 6032, 16455, 25198, 1	31393	5	15949, 6034, 1259, 28263, 15572
31477	6	26993, 6286, 28573, 26468, 25191	31489	7	25751, 8638, 26109, 19801, 17276
31513	7	25885, 31180, 3497, 8625, 30848	31573	5	9538, 29054, 8201, 6283, 22119
31657	5	13191, 8804, 707, 1978, 1	31729	7	897, 6796, 3763, 9119, 21548
31741	6	28021, 30268, 1763, 29588, 1473	31849	14	89, 30238, 3727, 1923, 2618
31873	11	28127, 5260, 23744, 11419, 18861	31957	2	10511, 30662, 30993, 3190, 1
31981	6	28342, 5469, 22009, 17267, 8270	32029	2	20933, 20672, 28581, 28660, 13717
32041	*	17327, 23373, 18572, 9946, 1	32077	2	26458, 31963, 1937, 3459, 2
32089	13	23181, 18292, 9863, 22795, 344	32173	5	32029, 11794, 18128, 23507, 28653
32233	5	3971, 25413, 21286, 27373, 21944	32257	15	21259, 15754, 28755, 31871, 20084
32341	2	7689, 21517, 16528, 23087, 2	32353	15	17032, 31364, 4663, 29327, 495
32377	5	987, 23134, 21325, 20045, 14768	32401	7	28403, 10090, 20091, 16699, 20180
32413	5	23918, 15232, 24649, 28229, 21225	32497	7	9535, 17752, 10493, 711, 3008
32533	2	5507, 25016, 11134, 7647, 1	32569	7	1077, 16354, 21223, 25919, 19490
32653	2	18413, 6118, 12002, 6279, 1	32713	5	8342, 16943, 10192, 18639, 18577
32749	2	19559, 2, 5896, 30267, 1	32761	*	13381, 8008, 21345, 10325, 20384

Table 11: Table of m, w, c, c', c'' for BIBD$(q, 12, 1)$ construction (q prime)

q	m	w	c	c'	c"	q	m	w	c	c'	c"
5413	2117	1224	3380	1941	4205	6073	2550	1842	5141	4875	1950
6337	1735	4824	4184	4073	3621	6469	2392	4992	64	6212	5069
6733	6378	6113	3475	4630	4821	6997	2492	4088	2164	1472	615
7129	2795	5879	2230	5669	11	7393	5394	1717	4266	6499	3661
7789	3822	233	6811	131	6852	8053	6744	3497	7598	3995	3883
8317	7077	1286	2936	7191	4486	8581	6263	424	398	318	3532
8713	5904	6175	1471	4408	4881	9109	5770	3120	5282	458	1019
9241	7962	9074	8415	5595	1745	9769	4461	9507	1557	324	1054
9901	6181	99	64	2204	6095	10429	6208	2414	1751	8331	10309
10957	8942	3145	4014	7721	10310	11353	8040	2307	413	10442	2232
11617	10190	9023	7271	8981	5420	12277	7847	11877	8192	6044	10550
12409	4841	9597	9833	5210	3068	12541	11585	10157	11102	7160	7059
13597	12373	4429	5256	10396	2739	13729	5476	7019	10912	11558	12736
14389	7105	13956	16	12102	5801	14653	3546	3693	7641	9119	2268
15313	8635	6819	6148	8085	875	15973	12061	1034	4771	6347	5908
16369	13275	338	11785	9786	6172	16633	752	4010	2100	7929	14216
17029	8861	5519	2782	757	9953	17293	7309	17161	10772	11099	354
18217	17856	8397	343	8847	9126	18481	16454	14188	5522	7092	9654
19009	7663	1625	529	7438	4112	19141	2425	10002	128	7791	12756
19273	14875	17533	15625	17315	587	19801	4142	2184	9251	19613	19387
20593	13610	143	1520	14365	3129	20857	8688	2047	10	16830	1720
21121	11634	13952	1704	16841	15698	21517	4770	2190	18424	2063	6494
21649	5358	14609	6130	19529	16194	22441	559	20414	17196	5967	7782
22573	15853	688	1296	4175	14805	23497	14115	23231	10478	21327	10548
23629	11971	20558	2048	9757	15004	23761	16962	14985	16122	19830	8136
23893	2208	5681	10368	6725	23602	24421	15074	5562	19965	17054	3656
25609	22883	11961	22243	24951	5228	25741	15895	24210	12965	17830	22885
25873	21339	23045	855	1200	4209	27061	26910	26896	256	1861	20151
27457	16675	23868	27290	21883	23298	28513	463	12370	15625	290	3950
28909	2499	10917	512	25493	24225	29173	22565	9972	4	18046	8170
29437	14250	21777	8	24758	27785	29569	614	13058	289	3578	28764
29833	15989	22037	625	10215	8611	30097	15065	22771	9709	24623	6781
30493	9170	21936	15006	13583	2737	30757	10533	15146	16766	13653	20901
31153	26022	30976	3104	13202	12205	32077	24294	15305	512	22778	15884
32341	22315	17796	2	19646	6292						

Table 12: Table of m, w, c, c', c'' for BIBD$(q, 12, 1)$ construction (q composite)

q	m	w	c	c'	c"
7921	$14x + 15$	$2x + 45$	$50x + 3$	$67x + 83$	$19x + 51$
11881	$89x + 13$	63	$29x + 100$	$92x + 69$	$76x + 41$
17161	$37x + 2$	$51x + 25$	$130x + 117$	$93x + 43$	$30x + 30$

Table 13: Table of $\gamma_1, \ldots, \gamma_5$ for RBIBD$(11q+1, 12, 1)$ construction

q	ω	$\gamma_1, \ldots, \gamma_5$	q	ω	$\gamma_1, \ldots, \gamma_5$
349	2	325, 34, 182, 341, 117	373	2	343, 364, 164, 105, 245
409	21	37, 118, 81, 266, 179	421	2	109, 334, 56, 401, 183
433	5	73, 243, 382, 155, 8	457	13	241, 416, 215, 376, 81
529	*	1, 239, 231, 284, 94	541	2	295, 503, 520, 315, 110
577	5	1, 353, 160, 129, 272	601	7	433, 75, 128, 23, 478
613	2	481, 375, 574, 149, 86	625	*	55, 494, 244, 399, 203
661	2	25, 519, 178, 191, 344	673	5	403, 641, 129, 578, 358
709	2	1, 47, 254, 154, 219	733	6	1, 267, 500, 587, 466
757	2	283, 341, 615, 152, 424	769	11	1, 560, 712, 525, 731
829	2	685, 8, 580, 449, 537	841	*	1, 423, 593, 86, 88
853	2	97, 112, 176, 99, 779	877	2	787, 700, 363, 773, 104
937	5	1, 111, 370, 305, 296	961	*	1, 640, 245, 525, 278
997	7	823, 34, 449, 207, 302	1009	11	103, 230, 803, 279, 610
1021	10	349, 717, 701, 1018, 506	1033	5	1, 166, 993, 401, 434
1069	6	1, 489, 836, 215, 616	1093	5	1, 459, 412, 284, 965
1117	2	1105, 155, 382, 315, 578	1129	11	1, 1100, 897, 934, 677
1153	5	163, 860, 531, 412, 947	1201	11	577, 977, 1048, 621, 200
1213	2	1, 220, 230, 105, 467	1237	2	1, 47, 110, 40, 1143
1249	7	1, 358, 1094, 527, 279	1297	10	1, 908, 1037, 10, 495
1321	13	565, 772, 831, 482, 1247	1369	*	1, 38, 681, 713, 292
1381	2	1, 424, 899, 1202, 1179	1429	6	1, 953, 21, 1180, 476
1453	2	1, 1409, 243, 106, 566	1489	14	1, 1336, 681, 1172, 731
1549	2	799, 886, 446, 539, 279	1597	11	1375, 976, 755, 1550, 1395
1609	7	1, 1096, 812, 1391, 879	1621	2	1549, 713, 1228, 9, 1046
1657	11	1, 1156, 1587, 818, 1451	1669	2	901, 569, 1292, 1342, 1083
1681	*	235, 933, 608, 1505, 652	1693	2	1, 206, 5, 724, 1413
1741	2	1, 1353, 358, 932, 779	1753	7	1, 1180, 1124, 1685, 1011
1777	5	1, 730, 620, 1157, 1047	1789	6	1, 1598, 1036, 125, 81
1801	11	1, 251, 273, 250, 1478	1849	*	1285, 1587, 346, 1193, 584
1861	2	547, 4, 1430, 131, 1125	1873	10	1, 527, 981, 170, 1222
1933	5	1, 1354, 1755, 743, 998	1993	5	37, 214, 914, 1259, 591
2017	5	1, 1131, 800, 1673, 316	2029	2	1981, 1899, 976, 644, 275
2053	2	1, 562, 1934, 443, 963	2089	7	1, 1804, 99, 101, 524
2113	5	1, 863, 15, 622, 1088	2137	10	1279, 44, 1535, 1732, 1227
2161	23	1, 273, 839, 2012, 574	2197	*	1, 220, 305, 93, 980
2209	*	211, 94, 1415, 476, 381	2221	2	529, 1709, 1204, 1280, 1791
2269	2	739, 1018, 755, 1917, 1736	2281	7	1, 1742, 1779, 1126, 1823
2293	2	1, 1217, 422, 1773, 2242	2341	7	1129, 1409, 1024, 591, 1982
2377	5	511, 831, 1786, 236, 917	2389	2	2347, 662, 2169, 1864, 95
2401	*	1, 2339, 796, 1473, 1730	2437	2	1, 1114, 2403, 1535, 2138
2473	5	1, 657, 238, 1718, 137	2521	17	1, 2470, 2297, 38, 1965
2557	2	1, 2374, 2381, 1778, 2085	2593	7	1, 1160, 2165, 1264, 963
2617	5	1, 686, 1630, 1485, 749	2677	2	1, 515, 994, 812, 279
2689	19	1267, 322, 965, 1946, 489	2713	5	1, 338, 27, 569, 1678
2749	6	1, 2080, 219, 1118, 1793	2797	2	1, 1853, 561, 1868, 2080
2809	*	1, 1406, 2625, 304, 587	2833	5	1, 2344, 1379, 1677, 1250
2857	11	931, 1943, 308, 1989, 1348	2917	5	1, 2537, 208, 1827, 2108
2953	13	1, 1269, 2447, 2620, 1268	3001	14	145, 1954, 2895, 467, 1874
3037	2	1, 2252, 2247, 989, 2050	3049	11	1, 2049, 1960, 2336, 377
3061	6	1, 1815, 2936, 827, 430	3109	6	1, 112, 1707, 2504, 467
3121	7	1, 2951, 2912, 1102, 81	3169	7	433, 1892, 827, 1312, 723
3181	7	2095, 443, 2019, 1304, 1582	3217	5	1, 2698, 2751, 2513, 2996
3229	6	1, 44, 2691, 143, 2440	3253	2	865, 1814, 1955, 2206, 2535
3301	6	1, 866, 1925, 291, 862	3313	10	1, 2769, 2860, 527, 1802
3361	22	1, 2420, 197, 2752, 2505	3373	5	1, 2925, 2231, 568, 722
3433	5	1, 2795, 99, 1486, 692	3457	7	1, 2901, 820, 1127, 2114
3469	2	1, 1493, 1160, 165, 2278	3481	*	1, 3100, 3461, 2889, 2918
3517	2	1807, 3003, 3452, 46, 1709	3529	17	1909, 2948, 1270, 1377, 2411
3541	7	1, 794, 640, 1179, 3197	3613	2	1, 748, 1940, 1949, 1257
3637	2	1, 789, 2549, 3556, 1964	3673	5	2083, 3320, 808, 1893, 3377
3697	5	175, 536, 765, 448, 2933	3709	2	1, 1593, 1421, 2644, 2354
3721	*	1, 124, 2361, 3713, 2252	3733	2	1537, 1287, 692, 2668, 1829
3769	7	1, 878, 2099, 1624, 3	3793	5	3457, 2751, 352, 2597, 1670
3853	2	3085, 2163, 2549, 1726, 3080	3877	2	1, 1790, 3167, 1215, 1678
3889	11	1, 3692, 2823, 3479, 3214	4021	2	1, 2558, 3052, 3009, 1247
4057	5	1945, 1264, 387, 1127, 3686	4093	2	25, 2, 1792, 2801, 1047
4129	13	1, 1828, 3255, 425, 632	4153	5	1, 1299, 1415, 3178, 500
4177	5	2545, 2, 323, 705, 2662	4201	11	3643, 3962, 3809, 2830, 531
4261	2	1, 2, 1343, 2452, 2145	4273	5	3457, 680, 1198, 513, 2855
4297	5	1, 2800, 3201, 2219, 2270	4357	2	1, 4, 3557, 615, 1982
4441	21	1, 2919, 122, 4385, 2986	4489	*	1, 2584, 1377, 3443, 2552
4513	7	4141, 2325, 2102, 1811, 4372	4549	6	1, 4282, 2957, 1934, 3411
4561	11	2647, 608, 2697, 4121, 4336	4597	5	1, 3735, 3238, 2390, 3239
4621	2	1, 3664, 3215, 999, 1940	4657	15	1, 2140, 1107, 3899, 788
4729	17	319, 916, 3968, 3737, 1515	4789	2	1, 2, 3285, 2801, 832
4801	7	4225, 3142, 2061, 3377, 2732	4813	2	1, 2734, 2792, 4737, 971
4861	11	1, 3892, 1193, 429, 4586	4909	6	1, 3657, 1960, 1982, 4511
4933	2	3385, 1445, 471, 310, 500	4957	2	949, 278, 1828, 255, 4457
4969	11	1, 4172, 2975, 3183, 3352	4993	5	1, 3094, 3896, 2951, 4479
5041	*	139, 2680, 2025, 2876, 173	5077	2	4363, 2, 1715, 4828, 1323
5101	6	1, 1317, 1769, 4634, 100	5113	19	1, 2973, 3764, 232, 5051
5197	7	3379, 3987, 4619, 494, 1990	5209	17	4837, 604, 4217, 2217, 2030
5233	10	1, 832, 968, 687, 5063	5281	7	1, 188, 4061, 2524, 2877
5329	*	1, 1184, 2555, 4029, 4156	5413	5	1, 2631, 4232, 1883, 2632
5437	5	1, 232, 3008, 3165, 2021	5449	7	1, 760, 1793, 404, 2787
5521	11	3937, 3692, 1384, 2339, 489	5557	2	1, 880, 4976, 1097, 345
5569	13	1, 190, 2948, 2235, 1055	5581	6	1, 586, 3513, 1067, 1694
5641	14	5485, 3910, 2991, 2630, 2939	5653	5	1, 3117, 1678, 1199, 2480
5689	11	1, 136, 1697, 639, 2282	5701	2	1, 3557, 188, 1006, 489
5737	5	1777, 4415, 4971, 1600, 5282	5749	2	1, 5661, 1186, 779, 1424
5821	6	1, 2685, 4540, 4892, 4709	5857	7	427, 4034, 5650, 1499, 5307
5869	2	1, 2, 2313, 4384, 4415	5881	31	2425, 1654, 686, 4481, 4497
5953	7	1, 1743, 2650, 3956, 1511	6037	5	1, 1485, 3440, 1175, 868
6073	10	1, 3622, 1227, 5987, 5090	6121	7	1, 124, 993, 3716, 413
6133	5	1, 891, 1270, 3170, 5099	6217	5	1, 404, 4385, 2271, 1006
6229	2	1, 2, 4199, 1240, 3321	6241	*	1, 80, 1593, 4937, 3154
6277	2	1, 2, 5824, 3267, 4649	6301	10	1, 1556, 4155, 4937, 3574
6337	10	1, 836, 2067, 4079, 280	6361	19	4855, 4796, 5253, 2434, 1289
6373	2	1, 1258, 2907, 4874, 881	6397	2	1, 6074, 4727, 3867, 1150
6421	6	1, 5661, 4244, 449, 2614	6469	2	1, 2, 3831, 3376, 563
6481	7	1, 4096, 1629, 2036, 3305	6529	7	1, 5768, 6208, 5997, 6263
6553	10	1, 392, 2873, 3322, 2559	6577	5	1, 898, 1622, 5031, 887
6637	2	3289, 4095, 2606, 712, 2129	6661	6	1, 1497, 4844, 4798, 3599
6673	5	1, 4136, 3071, 3357, 586	6709	2	847, 2, 4665, 5237, 5728
6733	2	1, 3572, 1216, 795, 683	6781	2	1, 1016, 5811, 352, 4763
6793	10	1, 892, 3839, 4125, 950	6829	2	1, 5482, 5324, 3597, 5585
6841	22	1, 5764, 3201, 3803, 5642	6889	*	1, 3446, 5283, 4175, 4906
6949	2	1, 128, 6017, 5026, 1473	6961	13	1, 6320, 2717, 6129, 3862
6997	5	1, 6928, 3905, 1863, 5594	7057	5	1, 1942, 4613, 1040, 4353
7069	2	6493, 845, 4448, 2997, 1372	7129	7	1543, 4108, 2876, 473, 5421
7177	10	1, 1964, 3736, 6153, 5831	7213	5	1, 3861, 1618, 4598, 617

Table 13: Table of $\gamma_1, \ldots, \gamma_5$ for RBIBD$(11q+1, 12, 1)$ construction (cont.)

q	ω	$\gamma_1, \ldots, \gamma_5$	q	ω	$\gamma_1, \ldots, \gamma_5$
7237	2	1, 2, 2134, 2537, 1911	7297	5	1375, 3458, 1930, 6143, 5889
7309	6	1, 4907, 898, 4700, 3753	7321	7	1, 5764, 4197, 3548, 5825
7333	6	1, 4137, 941, 4552, 5504	7369	7	7051, 1263, 1526, 2692, 4943
7393	5	1, 4498, 4943, 4778, 5529	7417	5	5653, 1490, 987, 7223, 5278
7477	2	1, 6592, 6767, 5025, 2450	7489	7	1, 1520, 5038, 3365, 4587
7537	7	1, 6267, 6215, 3376, 1574	7549	2	1, 1786, 2703, 2840, 1721
7561	13	1, 458, 1619, 3015, 6616	7573	2	3421, 2, 1959, 5519, 76
7621	2	1, 3088, 254, 3323, 945	7669	2	7645, 2, 6417, 7000, 4619
7681	17	3517, 3934, 569, 1628, 5679	7717	2	1237, 2283, 941, 3946, 7352
7741	7	1, 4955, 7148, 514, 7311	7753	10	1921, 6206, 5854, 3555, 1421
7789	2	1, 6980, 7233, 2237, 4318	7873	5	1, 4400, 4924, 7773, 3863
7921	*	1, 4049, 5536, 3638, 6423	7933	2	1, 7618, 737, 2504, 135
7993	5	1, 6844, 6851, 1808, 2697	8017	5	1, 7496, 1961, 5866, 3873
8053	2	5449, 2771, 3478, 2727, 6224	8089	17	5353, 1672, 795, 6782, 7181
8101	6	1, 4131, 484, 1592, 2225	8161	7	1, 6454, 2735, 338, 1653
8209	7	1, 4732, 2573, 1334, 2811	8221	2	5995, 2, 6443, 6358, 3243
8233	10	1, 6124, 1631, 4802, 6207	8269	2	1, 2, 8050, 4281, 4229
8293	2	1, 3910, 2162, 7491, 2729	8317	6	1, 7911, 500, 3197, 4180
8329	7	1903, 4246, 6188, 5243, 7797	8353	5	1, 868, 5858, 8261, 141
8377	5	199, 2813, 7341, 3032, 994	8389	6	1, 5235, 989, 5518, 6200
8461	6	1, 1490, 40, 2451, 467	8521	13	1, 5282, 1853, 4060, 2283
8581	6	1, 7358, 3861, 3814, 3305	8629	6	1, 3483, 8468, 1967, 1372
8641	17	1, 6502, 4083, 8450, 7907	8677	2	6565, 2, 730, 4833, 5801
8689	13	1, 8086, 647, 2078, 3951	8713	5	1, 7594, 5624, 5681, 1323
8737	5	1, 7118, 6209, 8649, 5122	8761	23	541, 3484, 200, 5069, 5607
8821	2	1, 2, 268, 8439, 3557	8893	5	1, 2793, 4130, 7822, 7505
8929	11	1, 4030, 7361, 5373, 5336	8941	6	1, 8007, 5180, 3484, 7109
9001	7	1, 5152, 6519, 7058, 449	9013	5	1, 2007, 1222, 725, 8396
9049	7	1, 4762, 5943, 3653, 8624	9109	10	1, 6604, 8396, 9039, 1835
9133	6	1, 1821, 7961, 1340, 6688	9157	6	1, 3038, 7731, 5471, 8092
9181	2	2557, 452, 6201, 4996, 3341	9241	13	1, 1724, 9197, 5931, 6820
9277	5	1, 4041, 7637, 748, 1184	9337	5	811, 3788, 3556, 4641, 491
9349	2	1, 6562, 1065, 446, 7751	9397	2	1, 2822, 550, 7071, 4415
9409	*	7711, 3332, 3358, 6725, 1899	9421	2	7621, 2, 9081, 4132, 797
9433	5	1, 9386, 658, 5933, 1203	9601	13	487, 4052, 9412, 7775, 759
9613	2	1489, 2, 5271, 310, 857	9649	7	1, 5240, 4587, 881, 6472
9661	2	1, 5992, 5252, 6617, 6465	9697	10	1, 3818, 3059, 339, 208
9721	7	1, 7846, 1421, 7413, 2000	9733	2	1, 4322, 1641, 8459, 6400
9769	13	1, 1600, 5612, 1019, 7893	9781	6	1, 8475, 7508, 2350, 4859
9817	5	9091, 4443, 5830, 5894, 8681	9829	10	8407, 6986, 4702, 3845, 1497
9901	2	9133, 2, 7073, 3873, 8800	9949	2	4159, 2, 8321, 993, 448
9973	11	5611, 3093, 9080, 598, 8957	10009	11	1, 7316, 1401, 2722, 3941
10069	2	1, 2, 9969, 2854, 4457	10093	2	1, 6584, 2373, 6526, 8309
10141	2	1, 4030, 6777, 5642, 1019	10177	7	1, 7840, 7364, 6545, 8559
10201	*	1, 5201, 6704, 6357, 682	10273	10	1, 7220, 4689, 6922, 7889
10321	7	1, 7754, 1462, 9093, 4103	10333	5	1, 3069, 3184, 7643, 3662
10357	2	6805, 6860, 9521, 6298, 10335	10369	13	1, 1288, 3632, 3383, 7683
10429	7	8239, 7118, 3449, 586, 9381	10453	5	325, 9182, 7942, 4151, 6171
10477	2	2185, 2, 3875, 6699, 6436	10501	2	1, 7214, 10145, 3610, 5847
10513	7	8317, 10300, 5978, 5567, 8601	10597	5	1, 8271, 7829, 572, 2584
10609	*	1, 9152, 7312, 1961, 3687	10657	7	1, 9501, 3500, 8213, 9712
10729	7	1, 10004, 5758, 3059, 10677	10753	11	1, 5499, 1528, 7184, 2345
10789	2	1, 9370, 1442, 9209, 9879	10837	2	1, 6070, 10661, 10358, 7971
10861	2	1, 2, 8195, 562, 1167	10909	2	1, 6082, 8987, 1832, 3765
10957	5	1, 1755, 10048, 9701, 386	10993	7	10471, 8420, 10227, 5674, 5447
11113	13	1, 393, 6844, 1613, 3098	11149	10	1, 4029, 7682, 10691, 58
11161	7	1, 550, 9392, 791, 3735	11173	5	1, 9987, 2978, 10048, 9473
11197	2	3067, 6250, 2300, 7337, 4035	11257	10	7303, 3272, 2537, 9382, 7803
11317	2	1, 10268, 11001, 9497, 6082	11329	7	1, 6988, 3428, 7509, 10997
11353	7	1765, 10726, 10013, 7802, 8931	11437	2	1, 7472, 2321, 6172, 6525
11449	*	1, 6927, 6152, 8884, 6581	11497	7	6043, 830, 3639, 7355, 6376
11593	5	1, 9860, 9599, 9148, 5787	11617	10	1, 6992, 11176, 1683, 7931
11677	2	11125, 8846, 11608, 2853, 725	11689	7	1, 8396, 5896, 2099, 1917
11701	6	1, 2165, 7785, 6208, 11336	11821	2	2023, 2487, 10178, 9839, 7072
11833	5	1, 1366, 3171, 11348, 9551	11881	*	1, 8360, 1149, 11051, 11644
11941	10	5575, 11150, 11782, 1389, 8141	11953	5	1, 11494, 2253, 9728, 8831
12037	5	3043, 5084, 3179, 2932, 6987	12049	13	1, 10048, 3993, 8717, 10760
12073	7	1, 1480, 41, 7857, 4634	12097	5	1, 50, 10761, 94, 8597
12109	6	1, 10953, 4043, 7312, 950	12157	2	1, 11924, 9970, 8391, 7163
12241	7	1, 1994, 832, 1737, 8717	12253	2	1, 11614, 10281, 10805, 10232
12277	2	1, 7462, 2415, 5987, 4106	12289	11	109, 4172, 11391, 8206, 11471
12301	2	1, 2, 11734, 2607, 11957	12373	2	1, 962, 7427, 9939, 11662
12409	7	10861, 6376, 2885, 11090, 10275	12421	7	4093, 9155, 6016, 9584, 4251
12433	13	1, 6362, 5907, 5302, 5231	12457	10	4225, 3400, 1905, 6356, 6401
12517	6	1, 10113, 8716, 10997, 3518	12541	14	4735, 3332, 10397, 6256, 9819
12553	5	1, 7852, 10019, 8361, 7826	12577	10	1, 4880, 4864, 5361, 8297
12589	2	1, 38, 657, 10762, 6965	12601	11	8671, 10718, 6358, 12431, 5145
12613	2	1, 6622, 12093, 7598, 10037	12637	2	1, 11134, 6218, 5085, 12263
12697	7	9967, 2468, 7595, 11542, 4005	12721	13	1, 1570, 9651, 857, 11528
12757	2	8617, 2, 7505, 10438, 4857	12769	*	1, 6497, 8522, 9357, 6184
12781	2	1, 3578, 4745, 5044, 8361	12829	2	1, 6944, 4, 10145, 12789
12841	21	1, 9590, 7655, 7551, 8812	12853	5	1, 11871, 5057, 12442, 1922
12889	13	1, 9982, 4725, 2915, 8180	12973	14	1, 237, 12238, 10664, 2567
13009	7	1, 2582, 3491, 8548, 4269	13033	5	11863, 5006, 6892, 5189, 5697
13093	6	1, 5931, 5932, 173, 7952	13177	5	1, 11770, 5831, 7682, 10761
13249	7	1, 11582, 609, 11218, 2711	13297	5	12055, 11794, 6509, 12243, 12086
13309	6	1, 639, 6293, 7172, 9580	13381	10	7855, 2330, 586, 3935, 7941
13417	5	1, 10640, 681, 11686, 10811	13441	11	1, 4600, 8375, 1125, 4022
13477	2	9085, 2, 12820, 8343, 9995	13513	5	1, 7415, 10028, 10354, 9627
13537	7	10243, 9542, 5855, 10665, 7444	13597	5	1, 723, 4037, 5978, 6196
13633	5	1, 2702, 5999, 238, 1515	13669	6	1, 113, 2650, 4083, 7742
13681	22	8575, 12352, 7767, 11159, 3458	13693	6	1, 7041, 518, 4930, 4037
13729	23	1, 13240, 7221, 2966, 12149	13789	7	5845, 10700, 6262, 513, 11525
13873	5	1, 3838, 12161, 12237, 10316	13921	7	1, 316, 7317, 5717, 8276
13933	2	1, 2800, 1613, 9728, 8433	14029	6	1, 13263, 11174, 1727, 5836
14149	6	1, 2205, 6644, 5062, 8819	14173	2	11845, 2, 11848, 7401, 9215
14197	11	8287, 11355, 7076, 1673, 13270	14221	2	7951, 10622, 10594, 4449, 4427
14281	19	1, 12674, 10919, 3375, 14218	14293	6	11359, 12825, 2248, 6026, 11999
14341	2	9931, 2, 1203, 7469, 3478	14389	2	1, 3196, 9395, 12291, 986
14401	11	1, 2966, 7383, 10816, 13601	14437	5	1, 1689, 13379, 14314, 8726
14449	22	1, 3956, 7576, 4841, 7965	14461	2	1, 12488, 2788, 2613, 8105
14533	2	4123, 2, 5745, 12790, 9881	14557	2	2107, 2, 706, 13655, 14451
14593	5	1, 376, 11415, 7646, 689	14629	2	13699, 3, 13300, 10985, 2024
14641	*	1, 8781, 9004, 101, 5048	14653	2	1, 3092, 3435, 8572, 5765
14713	5	7525, 13996, 6809, 237, 3998	14737	10	1, 10912, 1785, 2483, 9980
14797	2	1, 11866, 11438, 1371, 14111	14821	2	6193, 2, 6934, 14415, 9851
14869	2	1, 6308, 5381, 6189, 13132	14929	7	1, 11116, 5921, 2294, 507
15013	2	1, 11104, 809, 8612, 11577	15061	2	1, 13306, 2066, 9677, 8763
15073	5	1, 2342, 11585, 6730, 12975	15121	11	1, 952, 8651, 9320, 11667
15193	5	1, 3362, 7323, 5320, 5933	15217	10	8599, 12508, 11201, 171, 5960
15241	11	4711, 10768, 4910, 12345, 5711	15277	6	13699, 1173, 9416, 11015, 7966
15289	11	1, 2422, 10628, 7671, 15257	15313	5	1, 13526, 2855, 2313, 4306
15349	2	1, 2, 2357, 12945, 7654	15361	7	1, 9872, 9783, 1775, 3286

Table 13: Table of $\gamma_1, \ldots, \gamma_5$ for RBIBD$(11q+1, 12, 1)$ construction (cont.)

q	ω	$\gamma_1, \ldots, \gamma_5$	q	ω	$\gamma_1, \ldots, \gamma_5$
15373	2	1, 3592, 572, 11411, 9159	15493	5	1, 3033, 3436, 161, 10268
15541	6	1, 11810, 9189, 1666, 7157	15601	23	1, 5374, 12056, 15243, 1991
15625	*	1, 11714, 2919, 5783, 4030	15649	11	1, 2720, 13510, 12225, 13433
15661	2	1, 6602, 8909, 9430, 669	15733	6	1, 2955, 962, 3988, 7997
15817	5	1, 1052, 7240, 5391, 581	15877	5	1, 10094, 5013, 10931, 12538
15889	21	1735, 2925, 7442, 11459, 8176	15901	10	1, 9111, 2296, 6194, 8987
15913	5	10399, 10305, 4072, 2429, 2510	15937	7	1, 10641, 575, 11120, 2446
15973	7	5845, 5979, 2168, 5693, 3718	16033	5	4669, 11842, 6854, 13407, 10481
16057	7	1, 663, 15773, 12448, 8450	16069	2	1, 12688, 7739, 6704, 3597
16129	*	1, 9344, 7385, 598, 6441	16141	6	1, 8495, 315, 9118, 902
16189	2	1, 6140, 12076, 4419, 6041	16249	17	1, 11372, 14477, 448, 5493
16273	7	1, 4628, 2001, 184, 7301	16333	2	1, 14950, 15, 7325, 8666
16369	7	1, 2384, 15393, 5303, 9448	16381	2	1, 14, 6731, 4545, 6412
16417	10	1, 686, 10197, 10000, 2669	16453	2	1, 4082, 8164, 6879, 4229
16477	2	10039, 2, 13481, 2194, 10497	16561	7	1, 14656, 4364, 5571, 12119
16573	2	5293, 10796, 7295, 5428, 15999	16633	15	1, 1640, 14866, 8649, 281
16657	5	9163, 3088, 6429, 9098, 16067	16693	2	7279, 2, 885, 7145, 8518
16729	13	1, 4898, 2003, 13245, 6634	16741	6	1, 8661, 3158, 9869, 4018
16921	17	13813, 12920, 15892, 10071, 14831	16981	2	1, 8512, 13427, 6158, 8139
16993	10	1, 6796, 7508, 10817, 15981	17029	10	1, 8782, 3746, 1115, 3219
17041	7	1, 11060, 12281, 13300, 381	17053	2	3253, 2, 15754, 9861, 3029
17077	2	1, 6038, 10071, 5512, 7079	17137	5	1, 16760, 15760, 11249, 16635
17161	*	1, 14552, 3100, 743, 8763	17209	14	1, 12448, 10886, 6197, 2853
17257	5	1, 6658, 13539, 4748, 5783	17293	7	5863, 9489, 7738, 14282, 6011
17317	2	2353, 2, 322, 9443, 12657	17341	6	1, 5409, 14774, 208, 4139
17377	7	1, 8428, 3407, 1004, 15147	17389	2	1, 2, 2854, 16577, 3381
17401	11	1, 2228, 17015, 1138, 13593	17449	14	1, 15778, 3191, 13250, 63
17497	5	1, 7516, 3665, 7892, 9843	17509	2	1, 10294, 12896, 1955, 16605
17569	11	1, 15652, 4331, 9722, 549	17581	10	3955, 7910, 2661, 731, 4936
17713	7	1, 15436, 2990, 2715, 6371	17737	7	1, 4792, 11607, 7442, 2417
17749	2	10633, 2, 2669, 184, 5637	17761	19	1, 17426, 8999, 2062, 1533
17881	7	7687, 2056, 15212, 10773, 3515	17929	11	1, 16508, 9166, 3911, 8901
17977	5	1, 5576, 16018, 4529, 1977	17989	2	1, 2120, 2092, 14771, 12927
18013	2	1, 7430, 7739, 8205, 1990	18049	13	1, 10544, 8709, 8590, 16757
18061	6	1, 15915, 8926, 12020, 5873	18097	5	12709, 14114, 16283, 5379, 1930
18121	23	11089, 2092, 12461, 12758, 16659	18133	5	1, 10671, 6472, 11612, 9713
18169	11	1, 3140, 2303, 11002, 3147	18181	2	1, 14168, 7018, 15087, 12995
18217	7	1, 14410, 17294, 2303, 13581	18229	2	8533, 2, 11848, 15671, 4065
18253	5	1, 10185, 12074, 13384, 12155	18289	13	1, 14810, 10323, 13906, 17921
18301	6	1, 16839, 11828, 13163, 15250	18313	10	9547, 15884, 8722, 3425, 16113
18397	6	6601, 16486, 2271, 17315, 12674	18433	5	1, 6296, 14632, 4815, 5615
18457	5	1, 15548, 15118, 14975, 459	18481	13	6403, 3824, 1288, 15867, 5771
18493	2	10771, 3, 5978, 3592, 15569	18517	6	1, 1599, 18226, 4508, 6773
18541	6	1, 7767, 14762, 10523, 190	18553	5	1, 15734, 3514, 3593, 9369
18637	2	3661, 221, 7544, 14722, 3453	18661	10	1, 12338, 10119, 14843, 8206
18757	2	1, 1792, 7325, 18272, 3021	18769	*	1, 9521, 3028, 18177, 506
18793	5	1, 358, 3669, 18707, 18140	18913	7	511, 9290, 3166, 8963, 17445
18973	2	1, 380, 16336, 17909, 10023	19009	23	1, 15620, 5146, 14771, 14523
19069	2	1, 13610, 16895, 11932, 16029	19081	17	1, 10737, 1330, 5606, 14585
19141	2	1, 1912, 11381, 16544, 4419	19213	5	1, 10821, 12340, 13199, 8636
19237	2	1, 17590, 5793, 6881, 1502	19249	7	17905, 5222, 3, 7727, 3316
19273	5	18127, 2404, 6563, 285, 12404	19309	6	1, 15423, 4886, 9244, 15077
19321	*	1, 140, 13847, 14272, 14535	19333	2	1, 13490, 1024, 10773, 4211
19381	7	11677, 5315, 17973, 7216, 18158	19417	5	1, 18424, 2012, 4481, 18915
19429	6	5743, 8079, 3592, 12839, 9158	19441	13	1, 4264, 5153, 356, 14187
19477	6	5443, 6861, 8717, 14786, 6778	19489	19	5437, 15676, 3167, 19124, 8601
19501	2	1, 10852, 10424, 4043, 17937	19597	2	1, 3586, 17234, 15971, 1689
19609	13	9019, 3680, 13583, 6345, 11188	19681	11	6391, 7420, 8084, 4211, 5667
19717	2	1, 6140, 3171, 6964, 15107	19753	5	1, 11984, 18891, 178, 14657
19777	11	1, 8325, 2734, 14660, 17975	19801	13	1, 15560, 10457, 14836, 14133
19813	2	67, 2, 14626, 18389, 8979	19861	11	1, 5073, 11888, 12413, 3136
19993	10	1, 18056, 1965, 10355, 7594	20029	2	1, 18508, 18369, 18176, 15869
20089	7	6487, 9188, 11043, 10511, 13660	20101	6	1, 16719, 13568, 7127, 808
20113	10	3013, 9706, 6731, 16826, 15807	20149	2	2209, 17043, 2932, 14435, 14480
20161	13	1225, 19286, 7539, 1174, 15041	20173	2	16471, 5213, 12994, 3842, 3267
20233	5	1, 13595, 957, 19078, 12386	20269	2	6487, 2, 11812, 2315, 18153
20341	2	1, 11236, 893, 854, 10269	20353	5	1, 15032, 19793, 8007, 17332
20389	6	1, 14961, 18586, 14891, 17522	20509	2	1, 2, 15615, 15370, 95
20521	11	1, 9176, 17734, 12089, 12063	20533	2	835, 2, 837, 6701, 19036
20593	5	13255, 8690, 6741, 16883, 3874	20641	7	1, 11768, 6154, 725, 12561
20749	2	17659, 5456, 7127, 13558, 10533	20773	2	17143, 2, 20445, 10613, 12586
20809	7	1, 8751, 5450, 4061, 11416	20857	10	1, 3880, 3020, 6755, 9153
20929	7	8965, 12710, 14200, 4719, 5543	21001	11	1, 8192, 19427, 3243, 16906
21013	2	1, 19526, 13433, 6460, 13209	21061	7	12295, 18045, 7541, 20530, 6824
21121	19	17593, 1706, 15743, 1017, 14962	21157	2	17605, 2, 8231, 2164, 15663
21169	13	1, 18290, 2224, 3615, 17069	21193	11	1, 15081, 19984, 14969, 9062
21277	6	1, 14103, 3077, 20444, 21214	21313	5	11299, 10700, 5242, 18711, 10283
21397	2	985, 2, 18873, 8566, 15695	21433	5	1, 16850, 9263, 4066, 19983
21481	13	1, 9742, 212, 5277, 18437	21493	2	1, 5720, 6231, 4426, 2279
21517	5	1, 5829, 12400, 15401, 7460	21529	11	1, 10156, 1433, 16112, 9195
21577	5	18757, 12460, 4467, 9101, 12650	21589	2	4573, 2, 3029, 2158, 20817
21601	7	1, 16888, 6617, 16544, 21339	21613	2	1, 15104, 15370, 17169, 287
21649	14	1, 19786, 3135, 17198, 9737	21661	2	1, 14740, 15200, 12935, 6345
21673	10	11875, 14456, 3910, 21027, 18521	21757	5	14581, 5368, 16677, 21734, 13385
21817	7	1, 12268, 20645, 16611, 2696	21841	11	1, 20860, 19749, 17573, 3740
21937	7	1, 18436, 104, 5261, 5673	21961	17	1795, 2116, 14912, 2199, 20159
21997	7	14041, 11559, 19906, 12824, 15419	22093	6	1, 8541, 17957, 17282, 18574
22129	19	1, 4874, 9029, 5026, 18837	22153	5	1, 3850, 4011, 20510, 7019
22189	2	1, 7724, 16882, 17591, 11073	22201	*	1, 11249, 1624, 20144, 11739
22273	5	1, 19544, 13019, 1132, 6969	22369	11	1, 17672, 13006, 13037, 18555
22381	10	1, 16025, 8384, 9027, 17578	22441	14	1, 10280, 8356, 10463, 135
22453	5	1, 4383, 2236, 13640, 18659	22501	2	1, 13462, 12956, 11699, 17061
22549	2	1, 2, 10810, 5, 12015	22573	6	1, 11153, 20848, 20187, 16532
22621	2	1, 4522, 13749, 21800, 5771	22669	2	9931, 2, 9933, 19156, 3371
22717	2	2947, 1622, 11297, 20823, 16078	22741	7	1, 16178, 12965, 2451, 7354
22777	7	20209, 17721, 4240, 21428, 10949	22801	*	19897, 6080, 1936, 7493, 10293
22861	2	17731, 2, 4, 6929, 8571	22921	7	12757, 17522, 10425, 2110, 17369
22993	5	1, 18682, 20870, 17735, 15417	23017	5	1, 21578, 9340, 14171, 20913
23029	2	1, 8038, 18173, 4838, 5859	23041	11	12643, 19196, 13581, 1631, 9886
23053	2	15769, 2, 13373, 15964, 19737	23173	5	1, 14871, 3874, 5249, 19454
23197	2	17299, 2, 6154, 593, 4785	23209	31	1, 21868, 22772, 16211, 18885
23269	6	1, 22251, 9134, 21767, 10366	23293	5	1, 4199, 3183, 18302, 8554
23473	5	1069, 8873, 3759, 8386, 19310	23497	5	1, 1162, 18747, 13277, 2012
23509	2	1, 3712, 6069, 7544, 7571	23557	5	7423, 1220, 4049, 2043, 46
23581	6	1, 741, 9407, 17342, 17794	23593	5	14611, 3658, 7808, 2919, 15047
23629	2	1, 19670, 11559, 6641, 7978	23677	5	1, 4185, 1253, 23540, 6112
23689	11	4675, 14126, 16137, 8573, 1138	23761	7	7303, 10520, 21059, 8662, 23403
23773	5	1, 21261, 2458, 21065, 5036	23833	5	10393, 2325, 23708, 575, 22900
23857	5	1, 18464, 16655, 2025, 15886	23869	2	2707, 3, 20714, 7547, 14884
23893	5	6061, 19268, 22786, 17729, 5307	23917	2	1, 12592, 32, 11897, 9783
23929	7	1, 12862, 12245, 21656, 5739	23977	5	1, 4822, 15111, 19805, 21056
24001	14	1, 7892, 10533, 18928, 7883	24049	19	1, 5878, 8816, 19305, 12161
24061	10	3241, 2283, 9410, 568, 5447	24097	5	1, 3206, 21333, 22355, 4780

Table 13: Table of $\gamma_1, \ldots, \gamma_5$ for RBIBD$(11q+1, 12, 1)$ construction (cont.)

q	ω	$\gamma_1, \ldots, \gamma_5$	q	ω	$\gamma_1, \ldots, \gamma_5$
24109	2	13309, 2, 13312, 9659, 4323	24121	13	1, 9304, 9305, 6932, 18279
24133	6	1, 21789, 10330, 14027, 16652	24169	11	1, 2710, 4322, 18107, 11475
24181	17	22345, 5319, 3062, 21448, 5273	24229	2	1, 13942, 22437, 2726, 8567
24337	5	12877, 21209, 20643, 19924, 11810	24373	7	17359, 21441, 16814, 7222, 9125
24421	7	14677, 22095, 20212, 11639, 13538	24469	14	11395, 4557, 7960, 21890, 12587
24481	11	7729, 15842, 15413, 4138, 6429	24517	5	1, 22425, 12299, 19354, 1868
24649	*	1, 1106, 12166, 3785, 18213	24697	5	1, 15586, 23024, 6905, 18753
24709	2	1, 2, 21058, 227, 20259	24733	2	12193, 2, 11572, 23499, 22133
24781	2	1, 2, 1589, 7959, 15688	24793	5	1, 15028, 20913, 458, 12875
24841	14	1, 20692, 4937, 19959, 5678	24877	5	1, 10347, 23686, 10361, 12314
24889	11	5047, 21154, 1779, 3842, 3221	25033	5	14059, 12734, 16510, 5999, 18477
25057	5	9031, 18556, 2753, 1971, 15992	25117	5	2899, 21746, 11596, 23153, 10851
25153	10	1, 24922, 3375, 21299, 19274	25189	2	1, 23740, 14607, 16415, 4688
25237	2	1801, 2, 20931, 8464, 18839	25261	7	6979, 16671, 23614, 10757, 1676
25309	13	1369, 20054, 15556, 11981, 14307	25321	19	1, 19334, 11679, 15934, 17501
25357	2	1, 18664, 24315, 23834, 227	25453	2	1, 24296, 9753, 12940, 18605
25537	10	1, 24914, 24802, 22289, 17685	25561	11	12409, 9550, 8277, 13400, 1439
25609	7	11539, 6556, 19793, 21207, 4214	25621	10	1, 20891, 7552, 25305, 19868
25633	5	1, 15532, 21185, 11889, 9512	25657	5	1, 21400, 25167, 22160, 269
25693	2	1, 23828, 5614, 23361, 18143	25717	2	1, 4154, 3, 8602, 7499
25741	6	1, 8145, 12700, 18326, 25337	25801	7	1, 16388, 22000, 4937, 8853
25849	7	15697, 19568, 24689, 9052, 15837	25873	10	1, 2026, 14492, 23297, 24603
25933	2	1, 9742, 25316, 6887, 20073	25969	7	8401, 18556, 1473, 107, 8708
25981	11	1, 1203, 23690, 17764, 12377	26017	5	1, 24100, 21794, 2673, 6173
26029	6	1, 10095, 16577, 194, 14338	26041	13	7351, 6356, 23291, 195, 21544
26053	2	1, 4256, 18359, 23626, 19989	26113	7	1, 404, 20267, 18981, 6838
26161	13	1, 20210, 14039, 5128, 615	26209	11	1, 23402, 15921, 24382, 8369
26293	6	1, 24747, 2939, 17884, 7892	26317	6	1, 4257, 25868, 2219, 15304
26437	5	1, 13923, 5968, 8168, 10313	26449	7	10531, 11926, 10301, 21225, 13844
26497	5	1, 16004, 2128, 17751, 19769	26557	2	1, 7102, 6321, 5702, 17387
26569	*	1, 9184, 17289, 21242, 8309	26641	7	1, 8024, 958, 1053, 13607
26701	22	1, 14385, 14315, 11668, 19898	26713	10	6079, 18358, 4016, 2579, 16767
26737	10	1, 19196, 18586, 1823, 9927	26821	2	25639, 2, 6340, 12519, 5225
26833	5	1, 5486, 9711, 6077, 9610	26881	11	7219, 26152, 21080, 3201, 14627
26893	5	1, 1881, 17728, 5984, 10295	26953	7	1, 21808, 5421, 11930, 6965
27061	2	1, 2, 20200, 26255, 18741	27073	5	1, 11402, 27033, 1709, 10108
27109	7	23731, 5925, 5560, 18020, 15131	27241	17	9385, 16580, 25792, 19647, 22331
27253	2	5665, 22814, 2170, 3987, 9089	27277	6	1, 24993, 25462, 25349, 24878
27337	5	17539, 12754, 11330, 10427, 3483	27361	7	1, 21146, 23321, 2410, 23931
27397	2	1, 7960, 17090, 1851, 23765	27409	13	1, 22234, 3147, 11534, 10127
27457	7	8671, 12082, 2384, 20241, 14627	27481	7	1, 17672, 5032, 26709, 11057
27529	7	1, 8462, 18671, 226, 16473	27541	19	1, 7064, 16503, 4121, 3898
27673	11	1, 9219, 27218, 22577, 5944	27697	5	1, 9208, 6747, 3728, 25949
27733	2	1, 18860, 4637, 2458, 795	27793	5	1, 2662, 14237, 14540, 4293
27817	5	1, 9784, 9195, 16067, 20396	27889	*	1, 27508, 21287, 6669, 24338
27901	2	1, 2, 22265, 21927, 5626	27961	13	21625, 25168, 25190, 10701, 27629
27997	5	1, 21287, 14716, 24776, 9003	28057	5	1, 16246, 6401, 2619, 23432
28069	7	18055, 7490, 16732, 21777, 245	28081	19	18115, 2258, 19444, 18771, 12197
28201	11	1, 11596, 21947, 10443, 6530	28297	5	1, 140, 8865, 22889, 24238
28309	2	1, 20812, 11993, 13905, 10034	28393	15	21415, 1509, 11830, 13439, 21452
28429	2	1, 2, 5307, 28360, 6011	28477	2	22141, 14458, 3651, 2723, 3554
28513	5	1, 21700, 12911, 10676, 26637	28537	5	27301, 27338, 6747, 12077, 22198
28549	2	1, 2, 22275, 20693, 27376	28561	*	1, 19040, 3376, 3545, 4653
28573	2	20809, 3485, 28118, 18711, 23020	28597	2	14131, 2, 16809, 24287, 27592
28621	13	17455, 4065, 22474, 5639, 3656	28657	5	1, 5584, 6933, 26156, 6371
28669	6	1, 6525, 1655, 1708, 7724	28729	22	1, 11522, 40, 25769, 18447
28753	10	1, 21868, 3621, 19109, 18422	28789	7	23095, 16149, 4730, 18340, 21605
28813	2	13957, 7348, 20667, 13148, 22241	28837	2	1, 8854, 8145, 12605, 10928
28909	2	26773, 3, 20152, 1832, 22025	28921	11	1, 4856, 13097, 16474, 1137
28933	2	1, 26564, 23103, 12556, 21173	29017	5	1, 6880, 4310, 18863, 17817
29077	2	1, 11452, 2727, 6890, 4745	29101	2	7375, 9670, 5180, 16611, 4937
29137	5	5293, 2180, 14559, 19606, 17627	29173	2	20887, 2, 4, 24461, 5061
29209	7	403, 21484, 2462, 18407, 18285	29221	2	1, 2, 4534, 1047, 20087
29269	6	1, 18399, 26579, 25810, 2	29389	2	1, 2, 20200, 23483, 25821
29401	13	14299, 25006, 2, 20847, 19511	29437	2	1, 9188, 24633, 22078, 8345
29473	5	1, 3988, 16383, 11675, 15206	29569	17	1, 27170, 10473, 19139, 12706
29581	10	18955, 8330, 6263, 20703, 5272	29629	7	23413, 603, 25804, 27842, 9329
29641	7	29125, 1718, 13785, 28043, 26842	29761	17	17371, 12880, 27987, 17924, 22451
29833	5	1, 14360, 18965, 25312, 8691	29881	7	3409, 4586, 27646, 16509, 23471
29917	2	1, 25840, 2355, 20915, 16970	29929	*	1, 15137, 13312, 22868, 13305
29989	2	6793, 2, 26374, 18707, 7575	30013	2	1, 13552, 8735, 746, 23079
30097	10	1, 10612, 5120, 24813, 2081	30109	2	1, 7946, 11441, 5241, 21178
30133	5	1, 15627, 134, 8446, 28847	30169	7	1, 10952, 1109, 11326, 11325
30181	2	5329, 3646, 7493, 28820, 15567	30241	11	1, 10472, 22121, 6118, 5163
30253	2	1, 5410, 5631, 19838, 28133	30313	5	1, 21214, 24326, 28929, 20459
30469	2	1, 7562, 3, 317, 15508	30493	6	1, 27863, 22742, 22708, 4869
30517	2	1, 24430, 911, 11415, 4184	30529	13	11581, 7400, 30034, 6303, 18545
30553	5	27091, 13934, 26147, 16126, 28827	30577	5	1, 4558, 19139, 27513, 5126
30637	2	1, 11954, 26878, 6953, 20757	30649	7	1, 11572, 7844, 14697, 17549
30661	2	1, 17066, 22055, 9538, 18693	30697	10	1, 19762, 15584, 26675, 9249
30757	5	1, 24123, 24124, 21125, 16208	30781	2	1, 15046, 13817, 27897, 13208
30817	5	469, 14228, 16143, 18785, 24712	30829	2	1, 4270, 3, 14564, 28931
30841	7	9187, 3944, 18725, 16731, 15922	30853	2	1, 3490, 13961, 24177, 7964
30937	15	1, 7018, 29369, 23409, 21692	30949	10	1, 22959, 25781, 17104, 24860
31033	10	1, 20944, 20945, 20439, 7022	31069	2	1, 2, 10031, 11650, 2331
31081	13	1, 22708, 2792, 13877, 5043	31153	10	1, 3080, 14608, 18689, 15159
31177	7	14275, 1383, 20852, 11170, 16013	31189	13	1, 20583, 15280, 15434, 12203
31237	6	1, 15183, 15184, 17729, 266	31249	23	1, 20, 17470, 12605, 9525
31321	7	5869, 12314, 29015, 9321, 6490	31333	5	1, 11637, 14282, 30040, 24449
31357	2	1, 6032, 6431, 8986, 20265	31393	5	1, 15572, 12112, 30465, 29177
31477	6	1, 25191, 4562, 28817, 31414	31489	7	1, 8638, 16187, 3416, 30051
31513	7	1, 31180, 16532, 21263, 14661	31573	5	1, 22119, 27880, 9221, 23102
31657	5	1, 8804, 6497, 14445, 31498	31729	7	1, 21548, 20691, 15862, 5345
31741	6	1, 1473, 2948, 2609, 3358	31849	14	1135, 30238, 30255, 4484, 8837
31873	11	1, 18861, 17746, 6104, 6275	31957	2	1, 30662, 18111, 2092, 671
31981	6	1, 8270, 29181, 14213, 19636	32029	2	13717, 20672, 25623, 13918, 11081
32041	*	1, 23373, 15395, 1816, 18506	32077	2	31963, 2, 5, 15670, 4245
32089	13	1, 344, 20849, 27358, 3135	32173	5	1, 28653, 12083, 4312, 19106
32233	5	1, 5920, 4317, 17366, 11873	32257	15	1, 15754, 20342, 15825, 16475
32341	2	21517, 2, 26159, 12628, 20133	32353	15	1, 24669, 8318, 5614, 1955
32377	5	1, 23134, 29834, 25347, 15887	32401	7	1, 28948, 18788, 25473, 27107
32413	5	1, 21225, 10403, 12082, 19940	32497	7	1, 17752, 6429, 13907, 7016
32533	2	1, 25016, 16137, 598, 7679	32569	7	1, 19490, 28439, 244, 30399
32653	2	1, 6118, 8408, 3125, 13449	32713	5	18577, 16943, 21927, 31268, 32482
32749	2	1, 2, 22456, 11855, 14619	32761	*	1, 24934, 5475, 19655, 25154

Table 14: Table of 3960 possible exceptions in the PBD-closure of $\{7, 13\}$

19 25 31 37 43 55 61 67 73 79 97 103 109 115 121 127 133 139 145 151 157 163 181 187 193 199 205 211 223 229 235 241
247 253 265 271 277 283 289 313 319 325 331 349 355 361 367 373 391 397 403 409 415 433 439 445 451 457 475 481 487
493 499 505 523 529 535 541 565 571 577 583 607 613 619 625 643 649 655 661 667 685 691 697 703 709 715 727 733 739
745 751 769 775 781 787 793 799 805 811 817 823 829 835 853 859 865 871 877 895 901 907 913 919 925 937 943 949 955
961 979 985 991 997 1003 1021 1027 1033 1039 1045 1063 1069 1075 1081 1087 1111 1117 1123 1129 1135 1153 1159 1165
1171 1195 1201 1207 1213 1231 1237 1243 1249 1255 1273 1279 1285 1291 1297 1315 1321 1327 1333 1339 1357 1363 1369
1375 1381 1399 1405 1411 1417 1423 1435 1441 1447 1453 1459 1465 1489 1495 1501 1507 1525 1531 1537 1543 1549 1555
1567 1579 1585 1591 1609 1615 1621 1627 1633 1639 1645 1651 1657 1663 1669 1675 1693 1699 1705 1711 1717 1735 1741
1747 1753 1759 1765 1783 1789 1795 1801 1819 1825 1831 1837 1843 1861 1867 1873 1879 1885 1903 1909 1915 1921 1927
1945 1951 1957 1963 1969 1975 1987 1993 1999 2005 2011 2035 2047 2053 2077 2083 2089 2095 2119 2125 2131 2137 2155
2161 2167 2173 2179 2203 2209 2215 2221 2239 2245 2251 2257 2263 2281 2287 2293 2299 2305 2329 2335 2341 2347 2371
2377 2383 2389 2407 2413 2419 2425 2431 2455 2461 2467 2473 2491 2497 2503 2509 2515 2533 2539 2545 2551 2557 2575
2581 2587 2593 2599 2605 2623 2629 2635 2641 2665 2671 2677 2683 2701 2707 2713 2719 2725 2743 2749 2755 2761 2767
2785 2791 2797 2803 2809 2827 2833 2839 2845 2851 2875 2881 2887 2893 2917 2923 2929 2935 2959 2965 2971 2995 3001
3007 3013 3019 3037 3043 3049 3055 3061 3079 3085 3091 3097 3121 3127 3133 3139 3145 3163 3169 3175 3181 3187 3211
3217 3223 3229 3253 3259 3265 3271 3295 3301 3307 3313 3331 3337 3343 3349 3355 3373 3379 3385 3391 3397 3415 3421
3427 3433 3439 3457 3463 3469 3475 3481 3505 3511 3517 3523 3547 3553 3559 3565 3589 3595 3631 3637 3643 3649 3667
3673 3679 3685 3691 3715 3721 3727 3733 3757 3763 3769 3775 3799 3805 3811 3817 3841 3847 3853 3859 3883 3889 3895
3901 3919 3925 3931 3937 3943 3961 3967 3973 3979 3985 4003 4009 4015 4021 4027 4051 4057 4063 4069 4093 4099 4105
4111 4135 4141 4147 4153 4177 4183 4189 4195 4213 4219 4225 4231 4237 4255 4261 4267 4273 4279 4303 4309 4315 4321
4345 4351 4357 4363 4387 4393 4399 4405 4429 4435 4441 4447 4471 4477 4483 4489 4507 4513 4519 4525 4531 4555 4561
4567 4573 4591 4597 4603 4609 4615 4633 4639 4645 4651 4657 4681 4687 4699 4723 4729 4735 4741 4765 4771 4777 4783
4807 4813 4819 4825 4849 4855 4861 4867 4891 4897 4903 4909 4933 4939 4945 4951 4969 4975 4981 4987 5017 5023 5029
5035 5059 5065 5071 5077 5095 5101 5107 5113 5119 5137 5143 5149 5155 5179 5185 5191 5197 5203 5221 5227 5233 5239
5245 5269 5275 5281 5287 5311 5317 5323 5329 5347 5353 5359 5365 5371 5395 5401 5407 5413 5431 5437 5443 5449 5455
5479 5485 5491 5497 5515 5521 5527 5533 5539 5557 5563 5569 5575 5581 5605 5611 5617 5623 5647 5653 5659 5665 5689
5695 5701 5707 5725 5731 5737 5743 5749 5767 5773 5779 5785 5791 5815 5821 5827 5833 5857 5863 5869 5875 5899 5905
5911 5917 5935 5941 5947 5953 5959 5977 5983 5989 5995 6001 6025 6031 6037 6043 6067 6073 6079 6085 6109 6115 6121
6127 6151 6157 6163 6169 6193 6199 6205 6211 6235 6247 6253 6271 6277 6283 6289 6295 6313 6319 6325 6331 6337 6355
6361 6367 6373 6379 6397 6403 6409 6415 6421 6445 6451 6457 6463 6487 6493 6499 6505 6529 6535 6541 6547 6565 6571
6577 6583 6589 6607 6613 6619 6625 6631 6655 6661 6667 6673 6697 6703 6709 6715 6739 6745 6751 6757 6781 6787 6793
6799 6817 6823 6829 6835 6841 6865 6871 6883 6907 6913 6919 6925 6949 6955 6961 6967 6985 6991 6997 7003 7009 7033
7039 7045 7051 7075 7081 7087 7093 7117 7123 7129 7135 7153 7159 7165 7171 7177 7195 7201 7207 7219 7243 7249 7255
7261 7285 7291 7297 7303 7327 7333 7339 7345 7369 7375 7381 7387 7411 7417 7423 7429 7447 7453 7459 7465 7471 7489
7495 7501 7507 7513 7531 7537 7543 7549 7555 7579 7585 7591 7597 7621 7627 7633 7639 7663 7669 7675 7681 7705 7711
7717 7723 7741 7747 7753 7759 7765 7783 7789 7795 7807 7831 7837 7843 7849 7873 7879 7885 7891 7915 7921 7927 7933
7957 7963 7975 7999 8005 8011 8017 8035 8041 8047 8053 8059 8083 8089 8095 8101 8131 8137 8143 8167 8173 8179 8185
8209 8215 8221 8227 8251 8257 8263 8269 8293 8299 8305 8311 8335 8341 8347 8353 8371 8377 8383 8389 8395 8413 8419
8425 8431 8437 8461 8467 8473 8479 8503 8509 8515 8521 8545 8551 8557 8563 8587 8599 8605 8629 8635 8641 8647 8671
8677 8683 8689 8713 8719 8725 8731 8755 8761 8767 8773 8797 8803 8809 8815 8839 8845 8851 8857 8875 8881 8887 8893
8899 8917 8923 8929 8935 8941 8965 8971 8977 8983 9007 9013 9019 9025 9049 9055 9061 9067 9091 9097 9103 9109 9133
9139 9145 9151 9175 9181 9187 9193 9223 9229 9235 9259 9265 9271 9277 9295 9301 9307 9313 9319 9343 9349 9355 9385
9391 9397 9403 9427 9433 9439 9445 9469 9475 9481 9487 9511 9517 9523 9553 9559 9565 9571 9595 9601 9607 9613 9637
9643 9649 9655 9679 9685 9691 9697 9721 9727 9733 9739 9757 9763 9769 9775 9781 9799 9805 9811 9817 9823 9847 9853
9859 9865 9883 9889 9895 9901 9907 9931 9937 9943 9949 9973 9979 9985 9991 10015 10021 10027 10033 10057 10063
10069 10075 10099 10105 10111 10117 10135 10141 10147 10153 10159 10183 10189 10195 10201 10225 10231 10237 10243
10267 10273 10279 10285 10309 10315 10321 10327 10351 10357 10363 10369 10393 10399 10405 10411 10435 10441 10447
10453 10471 10477 10483 10489 10495 10513 10519 10525 10531 10537 10561 10567 10573 10579 10603 10609 10615 10621
10645 10651 10657 10687 10693 10699 10705 10729 10735 10741 10747 10771 10783 10789 10813 10819 10825 10831 10855
10861 10867 10873 10897 10903 10909 10915 10939 10945 10951 10957 10975 10981 10987 10993 10999 11017 11023 11029
11035 11041 11065 11071 11077 11083 11107 11113 11119 11125 11149 11155 11161 11167 11191 11197 11203 11209 11233
11239 11245 11251 11275 11281 11287 11293 11317 11323 11329 11335 11359 11365 11371 11377 11407 11413 11419 11443
11449 11455 11461 11485 11491 11497 11503 11527 11533 11539 11545 11569 11575 11581 11587 11611 11617 11623 11629
11647 11653 11659 11665 11671 11695 11701 11707 11713 11737 11743 11749 11755 11779 11785 11791 11797 11815 11821
11827 11833 11839 11863 11869 11875 11881 11905 11911 11917 11923 11947 11953 11959 11965 11989 11995 12001 12007
12031 12037 12043 12049 12073 12079 12085 12091 12115 12121 12127 12133 12157 12163 12169 12193 12199 12205 12211
12217 12241 12247 12253 12259 12277 12283 12289 12295 12301 12325 12331 12337 12343 12367 12373 12379 12385 12415
12421 12427 12457 12463 12469 12499 12505 12511 12541 12547 12553 12583 12589 12595 12619 12625 12631 12637 12661
12667 12673 12703 12709 12715 12721 12745 12751 12757 12763 12787 12793 12799 12805 12829 12835 12841 12847 12871
12877 12883 12889 12913 12919 12925 12931 12967 12973 12997 13003 13009 13015 13045 13051 13057 13075 13081 13087
13093 13099 13123 13129 13135 13141 13165 13171 13177 13207 13213 13219 13225 13249 13255 13261 13267 13291 13297
13303 13309 13333 13339 13345 13351 13381 13387 13393 13417 13423 13435 13459 13465 13471 13477 13501 13507 13513
13519 13543 13549 13555 13561 13585 13591 13597 13603 13633 13639 13645 13669 13675 13681 13687 13711 13717 13723
13753 13759 13765 13771 13801 13807 13813 13837 13843 13849 13855 13879 13885 13891 13915 13927 13933 13939 13969
13975 13981 14005 14011 14017 14023 14047 14053 14059 14065 14089 14095 14101 14107 14131 14137 14143 14149 14179
14185 14191 14221 14227 14233 14257 14263 14269 14275 14293 14299 14305 14311 14317 14341 14347 14359 14383 14389
14395 14401 14431 14437 14443 14473 14479 14485 14509 14515 14527 14551 14557 14563 14569 14593 14599 14605 14611
14641 14647 14653 14683 14689 14695 14719 14725 14731 14737 14761 14767 14773 14779 14803 14809 14815 14821 14845
14851 14857 14863 14887 14893 14899 14905 14929 14935 14941 14947 14971 14977 14983 15013 15019 15025 15031 15055
15061 15067 15073 15097 15103 15109 15115 15139 15151 15157 15181 15187 15193 15199 15229 15235 15241 15265 15271
15277 15283 15307 15313 15319 15325 15349 15355 15361 15367 15391 15397 15403 15409 15433 15439 15445 15451 15481
15487 15493 15517 15523 15529 15535 15559 15565 15571 15577 15607 15619 15643 15649 15655 15661 15691 15697 15733
15739 15745 15775 15781 15787 15811 15817 15823 15829 15853 15859 15865 15871 15901 15907 15913 15949 15955 15985
15991 15997 16021 16027 16033 16039 16063 16075 16105 16111 16117 16123 16147 16159 16165 16189 16201 16207 16231
16243 16249 16279 16285 16291 16327 16333 16357 16369 16375 16399 16411 16417 16441 16453 16459 16483 16489 16495
16501 16525 16531 16537 16543 16573 16579 16585 16609 16615 16621 16627 16651 16657 16663 16669 16699 16705 16711
16747 16753 16777 16789 16795 16819 16825 16831 16837 16867 16873 16879 16903 16909 16915 16921 16945 16951 16957
16963 16987 16993 16999 17029 17041 17047 17071 17077 17083 17089 17113 17119 17125 17131 17155 17161 17167 17197
17209 17215 17239 17245 17251 17257 17281 17287 17293 17299 17323 17329 17335 17341 17371 17377 17383 17407 17413
17419 17425 17455 17461 17467 17497 17503 17509 17533 17539 17545 17581 17587 17593 17617 17623 17629 17635 17659
17665 17671 17677 17707 17713 17749 17755 17761 17785 17791 17803 17827 17833 17839 17845 17869 17881 17887 17911
17923 17929 17965 17971 17995 18001 18007 18013 18049 18055 18079 18085 18091 18121 18127 18133 18139 18163 18175
18181 18205 18217 18223 18247 18253 18259 18265 18289 18295 18301 18307 18337 18343 18349 18373 18379 18385 18391
18415 18421 18427 18433 18457 18463 18469 18475 18499 18511 18517 18541 18547 18553 18559 18583 18589 18595 18601
18625 18631 18637 18643 18667 18673 18679 18685 18709 18715 18721 18727 18751 18757 18763 18769 18793 18805 18811
18847 18853 18883 18925 18931 18973 19009 19015 19051 19057 19093 19177 19183 19219 19261 19591 19597 19603 19609
19633 19639 19645 19651 19687 19693 19717 19729 19735 19759 19765 19771 19777 19801 19807 19819 19843 19849 19855
19861 19885 19891 19897 19903 19927 19939 19945 19969 19981 19987 20011 20017 20023 20029 20053 20059 20065 20071
20101 20107 20113 20149 20155 20179 20185 20191 20197 20221 20227 20233 20239 20263 20269 20275 20305 20311 20317
20323 20347 20353 20359 20365 20395 20401 20407 20431 20437 20443 20473 20479 20485 20491 20521 20527 20533 20557
20569 20575 20599 20605 20611 20617 20641 20647 20653 20659 20683 20689 20695 20701 20725 20731 20737 20743 20767
20773 20779 20785 20809 20815 20821 20827 20851 20863 20869 20905 20911 20941 20947 20989 21025 21031 21067 21109
21115 21151 21157 21235 21277 21319 21361 21733 21745 21751 21781 21787 21817 21823 21829 21835 21859 21865 21871
21877 21901 21907 21913 21943 21949 21955 21961 21985 21991 22003 22027 22033 22039 22045 22069 22075 22081 22087
22111 22117 22123 22129 22159 22165 22171 22201 22207 22213 22237 22249 22279 22285 22291 22297 22321 22327 22333
22339 22363 22369 22375 22381 22405 22411 22417 22423 22453 22459 22489 22495 22501 22507 22531 22537 22543 22549
22573 22579 22585 22591 22627 22657 22669 22675 22699 22705 22711 22717 22747 22753 22783 22795 22801 22825 22831
22837 22843 22867 22873 22879 22885 22909 22921 22927 22951 22957 22963 22969 22999 23005 23011 23041 23047 23053
23077 23095 23119 23125 23131 23137 23161 23167 23173 23203 23215 23221 23251 23293 23299 23305 23335 23341 23347
23371 23377 23383 23389 23419 23425 23431 23467 23473 23497 23509 23515 23539 23545 23551 23587 23593 23623 23629
23635 23641 23665 23671 23677 23683 23707 23719 23749 23761 23767 23791 23803 23839 23845 23881 23887 23893 23917
23929 23935 23959 23965 23971 24001 24007 24013 24043 24055 24061 24085 24091 24097 24127 24133 24139 24169 24175
24187 24217 24223 24229 24259 24265 24271 24307 24313 24355 24379 24385 24391 24433 24469 24475 24481 24511 24517
24553 24589 24595 24601 24631 24637 24673 24685 24721 24805 24841 24847 24883 24925 24967 25057 25063 25099 25105
25141 25147 25153 25177 25189 25219 25231 25261 25273 25303 25309 25315 25345 25351 25393 25399 25429 25435 25441
25447 25471 25483 25489 25513 25525 25531 25561 25567 25573 25597 25609 25645 25651 25657 25687 25693 25723 25729
25735 25765 25771 25777 25807 25813 25819 25849 25861 25891 25897 25903 25933 25939 25945 25981 25987 26017 26029
26059 26065 26071 26101 26107 26113 26143 26149 26155 26185 26191 26197 26233 26239 26275 26281 26317 26323 26359
26371 26401 26407 26437 26449 26479 26485 26491 26497 26527 26569 26575 26611 26617 26623 26647 26653 26659 26695
26701 26707 26731 26743 26749 26785 26827 26869 26875 26905 26911 26941 26947 26953 26983 26995 27001 27025 27037

Table 14: Table of 3960 possible exceptions in the PBD-closure of $\{7, 13\}$ (cont.)

27043	27067	27079	27109	27115	27121	27127	27151	27163	27169	27193	27199	27205	27211	27247	27253	27277	27289	27319
27331	27337	27373	27409	27415	27421	27451	27463	27493	27499	27529	27535	27541	27547	27577	27613	27619	27625	27631
27661	27667	27697	27709	27745	27751	27793	27835	27865	27877	27919	27949	27961	27997	28039	28045	28087	28117	28123
28129	28159	28165	28171	28201	28213	28243	28255	28291	28333	28375	28417	28459	28465	28471	28507	28513	28537	28579
28585	28627	28669	28711	28747	28789	28999	29005	29167	29209	29215	29251	29293	29713	29719	29725	29755	29761	29845
29851	29893	29923	29935	29941	29965	29977	29983	30007	30019	30055	30097	30139	30145	30175	30181	30193	30217	30223
30229	30235	30259	30271	30277	30391	30397	30427	30469	30475	30481	30511	30553	30565	30637	30643	30685	30721	30727
30763	30805	30847	30895	30931	30937	30979	31021	31099	31105	31111	31141	31183	31189	31225	31231	31273	31309	31315
31351	31393	31399	31477	31519	31537	31561	31567	31573	31609	31615	31645	31651	31657	31663	31687	31693	31705	31729
31741	31771	31777	31819	31861	31897	31909	31939	31945	31951	31957	31987	31993	32023	32029	32035	32041	32065	32107
32113	32155	32161	32167	32191	32197	32203	32233	32239	32245	32251	32287	32293	32317	32329	32335	32359	32365	32371
32401	32407	32449	32455	32461	32485	32491	32497	32527	32533	32539	32569	32575	32581	32587	32611	32653	32659	32665
32695	32701	32749	32755	32779	32785	32791	32821	32827	32833	32863	32869	32875	32881	32905	32917	32923	32947	32953
32959	32989	32995	33031	33037	33079	33085	33091	33121	33127	33133	33157	33163	33169	33175	33199	33211	33253	33289
33331	33337	33373	33379	33415	33421	33457	33499	33505	33547	33589	33625	33631	33667	33673	33715	33757	33919	33961
34171	34213	34585	34591	34597	34603	34627	34633	34639	34669	34675	34681	34687	34711	34717	34723	34753	34759	34807
34813	34837	34849	34879	34885	34891	34927	34933	34963	35005	35011	35017	35047	35053	35059	35089	35095	35101	35131
35137	35179	35185	35191	35221	35227	35233	35257	35275	35299	35305	35311	35317	35341	35347	35383	35425	35431	35443
35467	35473	35479	35485	35509	35515	35521	35527	35551	35557	35593	35599	35677	35689	35695	35719	35725	35767	35773
35803	35809	35815	35821	35845	35851	35857	35863	35887	35893	35899	35929	35935	35941	35947	35971	35977	35983	35989
36013	36019	36025	36061	36067	36073	36097	36109	36139	36145	36151	36157	36181	36187	36193	36223	36241	36271	36277
36283	36307	36313	36319	36349	36355	36367	36391	36397	36433	36439	36445	36451	36481	36487	36493	36559	36565	36571
36601	36607	36613	36643	36649	36685	36691	36697	36703	36727	36775	36781	36787	36811	36817	36859	36865	36871	36901
36907	36949	36955	36979	36991	37021	37027	37033	37069	37075	37111	37117	37153	37159	37189	37195	37201	37231	37279
37327	37369	37411	37453	37489	37531	37579	37699	37705	37741	37747	37789	37825	37831	37873	37957	37993	38035	38077
38119	38287	38329	38377	38875	38917	38953	38959	38965	38971	39001	39007	39037	39049	39079	39091	39097	39121	39163
39169	39211	39217	39223	39247	39253	39259	39295	39301	39307	39331	39373	39385	39415	39421	39427	39433	39457	39469
39499	39505	39511	39517	39541	39547	39553	39559	39583	39589	39601	39631	39667	39679	39685	39709	39721	39751	39799
39805	39811	39841	39847	39877	39883	39889	39919	39925	39931	39961	39967	40003	40009	40045	40051	40057	40093	40099
40129	40135	40141	40171	40177	40213	40219	40231	40297	40309	40345	40381	40387	40393	40423	40429	40471	40477	40507
40513	40519	40549	40561	40591	40597	40639	40681	40717	40723	40729	40759	40771	40777	40801	40807	40813	40819	40843
40849	40861	40885	40891	40927	40975	40981	40987	41011	41017	41053	41059	41065	41071	41107	41137	41149	41185	41191
41221	41227	41263	41269	41305	41311	41317	41347	41359	41395	41401	41479	41485	41515	41521	41527	41557	41569	41599
41605	41611	41641	41647	41689	41731	41773	41809	41815	41851	41899	41905	41941	41947	41983	42025	42067	42073	42103
42109	42151	42157	42193	42229	42235	42241	42271	42277	42283	42319	42361	42367	42409	42445	42451	42481	42487	42523
42529	42535	42565	42571	42577	42607	42619	42649	42691	42697	42739	42751	42775	42787	42793	42829	42835	42859	42871
42877	42901	42919	42943	42961	42985	42991	42997	43003	43033	43039	43045	43075	43081	43087	43111	43117	43153	43159
43165	43171	43243	43249	43291	43297	43327	43333	43363	43369	43375	43381	43405	43411	43417	43447	43453	43459	43489
43501	43531	43537	43543	43579	43585	43621	43627	43633	43657	43669	43699	43711	43741	43747	43783	43825	43831	43837
43867	43873	43879	43915	43921	43957	43999	44005	44011	44041	44047	44077	44089	44125	44131	44167	44209	44245	44251
44257	44287	44293	44299	44329	44335	44371	44413	44545	44551	44587	44593	44635	44665	44677	44713	44719	44749	44755
44797	44803	44809	44833	44839	44845	44881	44887	44893	44917	44929	44959	44971	44977	45013	45091	45103	45133	45139
45175	45181	45187	45217	45301	45337	45349	45385	45391	45421	45427	45433	45505	45511	45589	45601	45631	45679	45685
45727	45757	45769	45805	45811	45841	45847	45883	45889	45895	45973	46015	46051	46057	46093	46099	46105	46111	46177
46219	46261	46267	46273	46303	46309	46351	46357	46363	46387	46399	46429	46441	46471	46513	46519	46555	46561	46597
46639	46645	46651	46681	46687	46693	46723	46729	46741	46765	46771	46777	46855	46861	46897	46939	46945	46951	46981
46987	47017	47023	47059	47065	47101	47149	47161	47185	47191	47197	47227	47233	47239	47269	47275	47287	47311	47353
47365	47407	47449	47485	47527	47533	47563	47569	47605	47647	47653	47659	47695	47737	47773	47779	47785	47821	47827
47833	47869	47899	48031	48067	48073	48079	48109	48115	48121	48151	48193	48235	48241	48247	48277	48289	48319	48331
48361	48367	48403	48445	48457	48487	48499	48535	48541	48571	48577	48589	48619	48625	48631	48655	48661	48667	48697
48709	48739	48745	48751	48787	48823	48829	48865	48877	48913	48949	48955	48961	48991	49033	49039	49117	49129	49159
49207	49213	49243	49285	49291	49327	49333	49369	49381	49417	49459	49501	49537	49579	49585	49627	49753	49795	49837
49879	50047	50425	50467	50971	51055	51187	51307	51343	51385	51391	51721	51763	51805	51859	51865	51889	51895	51907
51931	51991	52099	52111	52183	52195	52231	52267	52279	52309	52315	52363	52393	52447	52531	52561	52567	52603	52687
52735	52813	52831	52855	52939	52951	53065	53071	53107	53155	53191	53281	53323	53329	53359	53371	53407	53413	53455
53485	53533	53569	53611	53653	53659	53707	53749	53827	53833	53863	53911	53917	53953	53959	53995	54037	54079	54199
54205	54247	54289	54331	54421	54451	54499	54541	54619	54625	54661	54667	54709	54793	54829	54835	54841	54871	54883
54913	54919	54925	54955	54967	54997	55087	55093	55123	55129	55165	55171	55177	55207	55249	55291	55297	55333	55381
55417	55459	55465	55471	55501	55513	55543	55561	55591	55597	55645	55681	55711	55717	55753	55759	55765	55801	55807
55837	55879	55921	55969	56005	56011	56047	56089	56185	56227	56305	56347	56353	56389	56431	56515	56557	56599	56689
56731	56809	56851	56893	56935	56983	57025	57145	57277	57361	57445	57529	57571	57781	57949	57991	58027	58069	58117
58453	58705	58747	58831	58867	58873	58909	59245	59287	59497	60379	60433	60469	60499	60511	60583	60715	60751	60805
60835	60889	60931	61051	61057	61087	61171	61345	61351	61423	61507	61513	61555	61597	61639	61645	61675	61807	61849
61891	61975	61981	62011	62185	62227	62233	62437	62479	62815	63361	63571	63949	63991	64117	64579	64621	64627	64699
64873	64915	64951	65125	65167	65419	65455	65461	65539	65581	66007	66049	66097	66211	66637	66679	66931	67225	67477
67519	67681	67771	67813	67975	68059	68149	68227	68269	68311	68359	68401	68563	68695	68779	68815	68857	69031	69151
69319	69361	69403	69445	69529	69571	69613	69619	69655	69697	69739	69823	69829	69871	69913	69949	69991	70033	70075
70081	70165	70201	70207	70243	70285	70291	70339	70345	70417	70453	70459	70465	70471	70495	70501	70537	70621	70663
70669	70705	70711	70753	70789	70801	70807	70831	70915	70921	70957	70975	71005	71047	71053	71083	71089	71095	71101
71131	71173	71209	71257	71293	71299	71335	71353	71377	71389	71425	71431	71467	71479	71503	71551	71587	71593	71677
71713	71719	71797	71845	71887	71971	72007	72049	72055	72091	72133	72175	72181	72223	72343	72475	72511	72517	72559
72595	72637	72679	72727	72811	72889	72931	72937	72979	73021	73057	73099	73105	73141	73147	73183	73225	73231	73351
73357	73435	73441	73477	73561	73609	73651	73693	73771	73813	73819	73855	73945	73987	74029	74065	74239	74281	74443
74491	74527	74533	74575	74707	74827	74869	74911	74917	74995	75031	75079	75115	75121	75163	75169	75211	75289	75331
75373	75379	75415	75457	75499	75583	75589	75619	75703	75709	75751	75799	75913	75961	76087	76093	76171	76261	76303
76339	76345	76387	76471	76549	76585	76591	76627	76669	76681	76711	76765	76807	76885	76891	76927	76933	76963	77047
77089	77137	77143	77173	77179	77215	77221	77227	77233	77257	77341	77347	77353	77383	77431	77467	77509	77515	77521
77551	77557	77593	77599	77635	77641	77647	77677	77725	77761	77767	77803	77809	77821	77845	77851	77857	77863	77971
77977	78019	78031	78055	78061	78109	78139	78151	78181	78223	78229	78313	78391	78439	78445	78475	78523	78535	78559
78577	78643	78649	78691	78733	78811	78895	78901	78913	78943	78985	79039	79063	79111	79117	79159	79231	79237	79279
79315	79321	79399	79483	79531	79621	79735	79783	79867	79903	79915	80077	80119	80161	80209	80251	80287	80323	80371
80407	80413	80461	80467	80497	80575	80707	80749	80755	80791	80827	80911	80995	81049	81127	81163	81295	81331	81379
81385	81415	81499	81631	81637	81643	81667	81751	81799	81967	81973	82003	82009	82051	82255	82267	82441	82471	82513
82549	82567	82645	82687	82723	82765	82807	82813	82939	83059	83101	83311	83353	83401	83605	83737	84031	85999	86041
86047	86053	86167	86293	86341	86377	86587	86677	86839	86929	87049	87133	87175	87265	87343	87559	87727	87931	87937
88105	88111	88141	88147	88189	88351	88441	88447	88483	88603	88693	88813	88897	89029	89107	89191	89233	89485	89527
89575	89785	89821	90073	90115	90367	90661	90667	90751	90793	90871	91249	91291	91507	91837	91879	92845	93349	93643
94189	94567	94609	95113	95197	95365	95659	95785	96625	96919	98683								