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Bezrukov-Gronau Order is Not Optimal

Let N be the integers ≥ 0. For $n \in N$ let $V(n)$ be the vectors $\underset{\sim}{a} = (a_1, \dots, a_n)$ with $a_i \in N$, and $C(n)$ be the vectors with $a_i \in \{0,1\}$. For $1 \leq i \leq n$ let $\delta_i \underset{\sim}{a} \in V(n-1)$ be obtained from $\underset{\sim}{a}$ by deleting a_i. The shadow $\Delta\underset{\sim}{a}$ is $\{\delta_1\underset{\sim}{a}, \dots, \delta_n\underset{\sim}{a}\} \subseteq V(n-1)$. For $A \subseteq V(n)$ we put $\Delta A = \bigcup\{\underset{\sim}{a} \in A\}\Delta\underset{\sim}{a}$.

Example 1: Let $E = \{000,\ 100,\ 010,\ 001,\ 110,\ 101,\ 011,\ 111,\ 200\}$, $F = E\bigcup\{210,\ 201,\ 211,\}$, $G = E\bigcup\{020,\ 002,\ 210\}$, $H = \{00,\ 10,\ 01,\ 11,\ 20\}$ then $\Delta F = H\bigcup\{21\}$ but $\Delta G = H\bigcup\{02, 21\}$.

Now $V\underset{\sim}{a}$ is the i with $a_{i-1} > a_i \leq a_{i+1} \leq \dots \leq a_n$ (think of this as V shaped) and $\underset{\sim}{a}^* = \delta_{V\underset{\sim}{a}}\underset{\sim}{a}$. Given $\underset{\sim}{a},\ \underset{\sim}{b} \in V(n)$ let $\omega(\underset{\sim}{a},\ \underset{\sim}{b})$ be the last i with $a_i \neq b_i$. In [2] we made

Definition 1 *(V-order)* **We order $V(1),\ V(2),\ \dots$ in turn by $\underset{\sim}{a} < \underset{\sim}{b}$ if**

(i) $\underset{\sim}{a}^* < \underset{\sim}{b}^*$,

or

(ii) $\underset{\sim}{a}^* = \underset{\sim}{b}^*$ **and** $a_j < b_j$ **where** $j = \omega(\underset{\sim}{a},\ \underset{\sim}{b})$.

The first so many sets of an order is an **initial section** IS. Thus $E,\ F,\ H$in Exemple 1 are IS of V-order. We say an order is **optimal** if for every $A \subseteq V(n)$ we get $|\Delta A| \geq |\Delta I|$, where I is the IS of the order of $V(n)$ with $|A| = |I|$. Strehl and Winkelmann defined SW-order and conjectured it was optimal [3]. We support that conjecture. In [2] we showed that V and SW orders are the same. Further we gave T-order for $C(n)$, and proved that both V-order and T-order are optimal over $C(n)$.

In [1] Bezrukov and Gronau gave BG-order and claimed it was optimal. The G of Exemple 1 is shown as an IS of BG-order so their claim is false. Sadly their proof is not correct even over $C(n)$ where BG-order is the same as V-order.

References

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