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On common fixed point theorems for families of mappings

ABSTRACT. In this paper we prove some common fixed point theorems for families of mappings on metric spaces which are generalizations of the results due to Fisher [2, 3, 4], Kasahara [5], Kim and Leem [6], Kim, Kim, Leem and Ume [7], Ohta and Nikaido [8], Park and Rhoades [9, 10], Taskovic [12] and others.

KEY WORDS AND PHRASES. Common fixed point, fixed point, family of mappings, closed mapping, cluster point, metric space, compact metric space.

1 Introduction and preliminaries

Let ω, N denote the sets of nonnegative integers and positive integers, respectively. Let f, g, h and t be self mappings of a metric space (X, d) and $C_f = \{s : s : X \to X \text{ and } sf = fs\}$. For $x, y \in X$ and $A, B \subseteq X$, define $O_f(x) = \{f^n x : n \in \omega\}$, $O_f(x, y) = O_f(x) \cup O_f(y)$, $O_{f,g}(x) = \{f^i g^j x : i, j \in \omega\}$, $O_{f,g}(x, y) = O_{f,g}(x) \cup O_{f,g}(y)$, $\delta(A, B) = \sup\{d(a, b) : a \in A, b \in B\}$, $\delta(A, A) = \delta(A)$ and $\delta(x, B) = \delta(\{x\}, B)$. It is easy to verify that $\{f^n : n \in \omega\} \subseteq C_f$. For each $t \in [0, +\infty)$, $[t]$ denotes the largest integer not exceeding t. Let

$$\begin{aligned}
\Phi_1 = \{\varphi : \varphi : [0, +\infty) \to [0, +\infty) &\text{ is nondecreasing, continuous from the right} \\
&\text{and satisfies } \varphi(t) < t \text{ for } t > 0\}, \\
\Phi_2 = \{\varphi : \varphi : [0, +\infty) \to [0, +\infty) &\text{ is nondecreasing, upper semicontinuous from the} \\
&\text{right and satisfies } \varphi(t) < t \text{ for } t > 0\}, \\
\Phi = \{\varphi : \varphi : [0, +\infty) \to [0, +\infty) &\text{ is nondecreasing, upper semicontinuous and} \\
&\text{satisfies } \varphi(t) < t \text{ for } t > 0\}.
\end{aligned}$$

Clearly, $\Phi \subseteq \Phi_2$. Chang [1] and Wong [13] proved that $\Phi_2 \subseteq \Phi$ and $\Phi_1 = \Phi_2$. Therefore $\Phi_2 = \Phi$.

The Banach contraction principle has long been one of the most important tools in the study of nonlinear problems. Motivated by this fact, during the past three decades, there has grown an extensive literature devoted to sharper forms of the principle. Recently, Fisher [2, 3, 4], Kasahara [5], Kim and Leem [6], Kim, Kim, Leem and Ume [7], Otha and Nikaido [8], Park and Rhoades [9, 10], Taskovic [12] and others have established the existence of fixed and common fixed points for the following contractive type mappings:

$$\exists r \in [0,1) \text{ such that for all } x, y \in X\,, \quad d(fx, fy) \le r\delta(O_f(x,y)) \qquad \text{(Taskovic [12])}\,; \tag{1.1}$$

$$\exists k \in N \text{ and } r \in [0,1) \text{ such that for all } x, y \in X\,, \quad d(f^k x, f^k y) \le r\delta(O_f(x,y)) \qquad \text{(Ohta and Nikaido [8])}\,; \tag{1.2}$$

$$\exists k \in N \text{ and } r \in [0,1) \text{ such that for all } x, y \in X\,, \quad d(f^k x, g^k y) \le r\delta(O_{f,g}(x,y)) \qquad \text{(Kim and Leem [6])}\,; \tag{1.3}$$

$$\exists k \in N \text{ and } r \in [0,1) \text{ such that for all } x, y \in X\,, \quad d((fg)^k x, (fg)^k y) \le r\delta(O_{f,g}(x,y)) \qquad \text{(Kim and Leem [6])}\,; \tag{1.4}$$

$$\exists m, n \in N \text{ and } r \in [0,1) \text{ such that for all } x, y \in X\,, \quad d((fg)^m x, (fg)^n y) \le r\delta(O_{f,g}(x,y)) \qquad \text{(Kim, Kim, Leem and Ume [7])}\,; \tag{1.5}$$

$$\exists \varphi \in \Phi_1 \text{ such that for all } x, y \in X\,, \quad d(fx, f^2 y) \le \varphi(\delta(O_f(x, fy))) \qquad \text{(Park and Rhoades [9])}\,; \tag{1.6}$$

$$\exists \varphi \in \Phi_1 \text{ such that for all } x, y \in X\,, \quad d(gx, gy) \le \varphi(\delta(O_f(x,y))) \qquad \text{(Park and Rhoades [10])}\,; \tag{1.7}$$

$$\exists \varphi \in \Phi_2 \text{ such that for all } x, y \in X\,, \quad d(fx, fy) \le \varphi(\delta(O_f(x,y))) \qquad \text{(Kasahara [5])}\,. \tag{1.8}$$

Now we list contractive type conditions to be considered:

$$\exists \varphi \in \Phi, p, q, m, n \in \omega \text{ with } p+q, m+n \in N \text{ such that for all } x, y \in X\,, \quad d(f^p g^q x, f^m g^n y) \le \varphi(\delta(O_{f,g}(x,y)))\,; \tag{1.9}$$

$$\exists \varphi \in \Phi, m \in \{1,2\} \text{ such that for all } x, y \in X\,, \quad d(fx, f^m y) \le \varphi(\delta(O_f(x,y)))\,; \tag{1.10}$$

$$\exists p, q, m, n \in \omega \text{ with } p+q, m+n \in N \text{ such that for all } x, y \in X \text{ with } f^p g^q x \ne f^m g^n y\,,\ d(f^p g^q x, f^m g^n y) < \delta\Big(\bigcup_{h \in C_f \cap C_g} h O_{f,g}(x,y)\Big)\,; \tag{1.11}$$

$$\exists p, q, m, n \in \omega \text{ with } p+q, m+n \in N \text{ such that for all } x, y \in X \text{ with } f^p g^q x \ne h^m t^n y\,,\ d(f^p g^q x, h^m t^n y) < \delta\Big(\bigcup_{u \in C_{fg}} u O_{fg}(x), \bigcup_{v \in C_{ht}} v O_{ht}(y)\Big)\,. \tag{1.12}$$

It is easy to see that the following diagrams of implications hold:

$$(1.9) \Leftarrow (1.7) \Leftarrow (1.8) \Leftarrow (1.1) \Rightarrow (1.2) \Rightarrow (1.3) \Rightarrow (1.99)\,,$$
$$(1.4) \Rightarrow (1.5) \Rightarrow (1.9)\,,$$
$$(1.6) \Rightarrow (1.10) \Rightarrow (1.9) \Rightarrow (1.11)\,.$$

In this paper we establish several fixed and common fixed point theorems involving hypotheses weak enough to include a number of results due to Fisher [2, 3, 4], Kasahara [5], Kim and Leem [6], Kim, Kim, Leem and Ume [7], Ohta and Nikaido [8], Park and Rhoades [9, 10], Taskovic [12] and others as special cases.

The following lemmas were introduced by Kim, Kim, Leem and Ume [7] and Sing and Meade [11], respectively.

Lemma 1.1 [7]. *Let f and g be commuting mappings from a compact metric space (X,d) into itself. Assume that fg is closed. If $A = \bigcap_{n\in N}(fg)^n X$, then*

(i) $hA \subseteq A$ *for all* $h \in C_{fg}$,

(ii) $A = fA = gA \neq \phi$,

(iii) A *is a compact subset of* X.

Lemma 1.2 [11]. *Let $\varphi \in \Phi$. Then*

(i) $\lim\limits_{n\to\infty} \varphi^n(t) = 0$ *for all* $t > 0$,

(ii) $t = 0$ *provided that* $t \leq \varphi(t)$ *for some* $t \geq 0$.

2 Main results

Our main results are as follows.

Theorem 2.1 *Let f and g be commuting mappings from a metric space (X,d) into itself such that fg is closed. Suppose that there exists $u \in X$ such that the sequence $\{(fg)^i u\}_{i\in N}$ has a cluster point $w \in X$ for which* (1.9) *holds for all $x, y \in O_{f,g}(u,w)$, where $\delta(O_{f,g}(u,w)) < +\infty$. Then w is a common fixed point of f and g and*

$$d((fg)^i f^a g^b u, w) \leq \varphi^{\left[\frac{i}{k}\right]}(\delta(O_{f,g}(u)))$$

for all $i \in N$ and $a, b \in \{0,1\}$, where $k = \max\{p,q,m,n\}$.

Proof: For any $i, j, l, s, t \in \omega$, it follows from (1.9) that

$$\begin{aligned} d(f^{i+k+j}g^{i+k+l}u, f^{i+k+s}g^{i+k+t}u) &\le \varphi(\delta(O_{f,g}(f^{i+k-p+j}g^{i+k-q+l}u, f^{i+k-m+s}g^{i+k-n+t}u))) \\ &\le \varphi(\delta(O_{f,g}(f^{i+j}g^{i+l}u, f^{i+s}g^{i+t}u))) \\ &\le \varphi(\delta(O_{f,g}((fg)^i u)))\,. \end{aligned}$$

This implies that

$$\delta(O_{f,g}((fg)^{i+k}u)) \le \varphi(\delta(O_{f,g}((fg)^i u))) \tag{2.1}$$

for all $i \in \omega$. We claim that

$$d((fg)^i u, (fg)^{i+t}u) \le \varphi^{[\frac{i}{k}]}(\delta(O_{f,g}(u))) \tag{2.2}$$

for all $i, t \in N$. We can write $i = ck + l$ uniquely for some $c, l \in \omega$ with $l \le k-1$. Using (2.1),

$$\begin{aligned} d((fg)^i u, (fg)^{i+t}u) &\le \delta(O_{f,g}((fg)^{ck+l}u)) \\ &\le \varphi(\delta(O_{f,g}((fg)^{(c-1)k+l}u))) \\ &\le \cdots \\ &\le \varphi^c(\delta(O_{f,g}((fg)^l u))) \\ &\le \varphi^c(\delta(O_{f,g}(u)))\,. \end{aligned}$$

That is, (2.2) holds. Lemma 1.2 ensures that $\{(fg)^i u\}_{i\in N}$ is a Cauchy sequence and since it has a cluster point $w \in X$, so $w = \lim_{i\to\infty}(fg)^i u$. Note that fg is closed. Then

$$w = \lim_{i\to\infty}(fg)^i u = \lim_{i\to\infty} fg(fg)^i u = fgw\,. \tag{2.3}$$

For any $i, j, s, t \in \omega$, by (1.9) and(2.3) we obtain

$$\begin{aligned} d(f^i g^j w, f^s g^t w) &= d(f^{i+k}g^{j+k}w, f^{s+k}g^{j+k}w) \\ &\le \varphi(\delta(O_{f,g}(f^{i+k-p}g^{j+k-q}w, f^{s+k-m}g^{t+k-n}w))) \\ &\le \varphi(\delta(O_{f,g}(w))) \end{aligned}$$

which implies that

$$\delta(O_{f,g}(w)) \le \varphi(\delta(O_{f,g}(w)))\,.$$

It follows from Lemma 1.2 that $\delta(O_{f,g}(w)) = 0$. That is, $w = fw = gw$.

In view of (2.1) we have

$$\begin{aligned} d((fg)^i f^a g^b u, (fg)^{i+t}u) &\le \delta(O_{f,g}((fg)^i u)) \\ &\le \varphi(\delta(O_{f,g}(fg)^{i-k}u)) \\ &\le \varphi^{[\frac{i}{k}]}(\delta(O_{f,g}(u))) \end{aligned}$$

for all $i, t \in N$ and $a, b \in \{0, 1\}$. Letting t tend to infinity we get

$$d((fg)^i f^a g^b u, w) \leq \varphi^{\left[\frac{i}{k}\right]}(\delta(O_{f,g}(u)))$$

for all $i \in N$ and $a, b \in \{0, 1\}$. This completes the proof.

Remark 2.1 Theorem 2.1 includes Theorems 3 and 4 of Kim and Leem [5], Theorems 1 and 2 of Park and Rhoades [10] as apecial cases.

From Theorem 2.1 we immediately have

Theorem 2.2 *Let f and g be commuting mappings from a bounded complete metric space (X, d) into itself such that fg is closed. Suppose that (1.9) holds. Then f and g have a unique common fixed point $w \in X$ and*

$$d((fg)^i f^a g^b x, w) \leq \varphi^{\left[\frac{i}{k}\right]}(\delta(O_{f,g}(x)))$$

for all $x \in X$, $i \in N$ and $a, b \in \{0, 1\}$, where $k = \max\{p, q, m, n\}$.

Remark 2.2 Theorem 2.2 includes Theorem 2.1 of Kim, Kim, Leem and Ume [6] as a special case.

Corollary 2.1 *Let f be a closed mapping from a metric space (X, d) into itself. Suppose that there exists $u \in X$ such that the sequence $\{(fg)^i u\}_{i \in N}$ has a cluster point $w \in X$ for which the following*

$$d(f^p x, f^m y) \leq \varphi(\delta(O_f(x, y))) \tag{2.4}$$

holds for all $x, y \in O_f(u, w)$, where $\delta(O_f(u, w)) < +\infty$, $\varphi \in \Phi$ and $p, m \in N$. Then f has a fixed point w and satisfies

$$d(f^i u, w) \leq \varphi^{\left[\frac{i}{k}\right]}(\delta(O_f(u)))$$

for all $i \in N$, where $k = \max\{p, m\}$.

Remark 2.3 In case $\varphi(t) = rt$ and $p = m$, Corollary 2.1 reduces to a result which extends Theorem 3 of Ohta and Nikaido [7].

The following result reveals that the condition that T be closed is unnecessary if $p = 1$ and $m \in \{1, 2\}$.

Theorem 2.3 *Let f be a mapping from a metric space (X, d) into itself. Suppose that there exists $u \in X$ such that the sequence $\{f^i u\}_{i \in N}$ has a cluster point $w \in X$ for which (1.10) holds for all $x, y \in O_f(u, w)$, where $\delta(O_f(u, w)) < +\infty$. Then w is a fixed point of f and satisfies*

$$d(f^i u, w) \leq \varphi^{\left[\frac{i}{m}\right]}(\delta(O_f(u))) \tag{2.5}$$

for all $i \in N$.

Proof: It follows from the proof of Theorem 2.1 that

$$\delta(O_f(f^{i+m}u)) \leq \varphi(\delta(O_f(f^iu))) \tag{2.6}$$

for all $i \in \omega$,

$$d(f^iu, f^{i+t}u) \leq \varphi^{\left[\frac{i}{m}\right]}(\delta(O_f(u))) \tag{2.7}$$

for all $i, t \in N$ and $w = \lim\limits_{i\to\infty} f^iu$. Letting t tend to infinity in (2.7), we easily conclude that (2.5) holds.

For every $\varepsilon > 0$ there exists an integer $k > 2m$ such that $i > k - m$ implies $d(f^iu, w) < \varepsilon$. For any $n, p \in N$ with $p > k$, by (1.9) we have

$$\begin{aligned} d(w, f^nw) &\leq d(w, f^pu) + d(f^pu, f^nw) \\ &\leq \varepsilon + \varphi(\delta(O_f(f^{p-1}u, f^{n-m}w))) \\ &\leq \varepsilon + \varphi(\max\{2\varepsilon, \delta(O_f(w)) + \varepsilon\}) \end{aligned}$$

which implies that

$$\delta(w, O_f(w)) \leq \varepsilon + \varphi(\max\{2\varepsilon, \delta(O_f(w)) + \varepsilon\}) \,.$$

Thus we have

$$\begin{aligned} \delta(w, O_f(w)) &\leq \limsup_{\varepsilon\to 0}\{\varepsilon + \varphi(\max\{2\varepsilon, \delta(O_f(w)) + \varepsilon\})\} \\ &\leq \limsup_{\varepsilon\to 0} \varphi(\max\{2\varepsilon, \delta(O_f(w)) + \varepsilon\}) \\ &\leq \varphi(\delta(O_f(w))) \,. \end{aligned}$$

That is,

$$\delta(w, O_f(w)) \leq \varphi(\delta(O_f(w))) \,. \tag{2.8}$$

For $m = 1$, by (2.6) and (2.8) we have

$$\delta(O_f(w)) = \max\{\delta(w, O_f(fw)), \delta(O_f(fw))\} \leq \varphi(\delta(O_f(w))) \,.$$

Lemma 1.2 ensures that $\delta(O_f(w)) = 0$. Hence $w = fw$.

For $m = 2$, by (2.6), (1.10) and(2.8) we get

$$\begin{aligned} \delta(O_f(w)) &= \max\{\delta(w, O_f(fw)), \delta(fw, O_f(f^2w)), \delta(O_f(f^2w))\} \\ &\leq \max\{\varphi(\delta(O_f(w))), \sup_{t\in\omega} \varphi(\delta(O_f(w, f^tw))), \varphi(\delta(O_f(w)))\} \\ &= \varphi(\delta(O_f(w))) \,. \end{aligned}$$

It follows from Lemma 1.2 that $\delta(O_f(w)) = 0$. That is, $w = fw$. This completes the proof.

Remark 2.4 Theorem 2.3 extends, improves and unifies Theorem 2 of Park and Rhoades [8] and Theorem 1 of Kasahara [4].

Question 2.1 Does Theorem 2.3 hold for $m \geq 3$?

Theorem 2.4 *Let f and g be commuting mappings from a compact metric space (X,d) into itself such that fg is closed. If* (1.11) *holds, then f and g have a unique common fixed point $w \in X$. Moreover, $w = hw$ for all $h \in C_f \cap C_g$.*

Proof: Let $A = \bigcap_{i \in N}(fg)^i X$. Lemma 1.1 implies that $A = fA = gA \neq \phi$. We assert that $A = \{w\}$ for some $w \in X$. Otherwise $\delta(A) > 0$. It follows from the compactness of A that $\delta(A) = d(u,v)$ for some $u, v \in A$. Obviously there exist $x, y \in A$ such that $u = f^p g^q x$, $v = f^m g^n y$. By (1.11) and Lemma 1.1 we conclude that

$$\begin{aligned}
\delta(A) &= d(f^p g^q x, f^m g^n y) \\
&< \delta\Big(\bigcup_{h \in C_f \cap C_g} hO_{f,g}(x,y)\Big) \\
&\leq \delta\Big(\bigcup_{h \in C_f \cap C_g} hA\Big) \\
&\leq \delta(A)
\end{aligned}$$

which is a contradiction. Thus A is a singleton and $A = \{w\}$ for some $w \in X$. Therefore $w = fw = gw$. That is, w is a common fixed point of f and g.

Now suppose that f and g have a second common fixed point u. Then $u = (fg)^i u$ for all $i \in N$. This implies that $u \in A = \{w\}$. That is, $u = w$. This proves the uniqueness of w, which implies that $w = hw$ for all $h \in C_f \cap C_g$. This completes the proof.

Remark 2.5 Theorem 4 of Fisher [1], Theorem 5 of Fisher [2] and Theorem 2 of Fisher [3] are special cases of Theorem 2.4.

Theorem 2.5 *Let f, g, h and t be self mappings of a compact metric space (X,d) such that* (1.12) *holds. If fg, ht are closed and $f \in C_g$, $h \in C_t$, then f, g, h and t have a unique common fixed point $w \in X$. Moreover, $w = uw = vw$ for $u \in C_{fg}$ and $v \in C_{ht}$.*

Proof: Put $A \bigcap_{i \in N}(fg)^i X$ and $B = \bigcap_{i \in N}(ht)^i X$. From Lemma 1.1 it follows that

$$A = fA = gA \neq \phi\,, \quad B = hB = tB \neq \phi\,,$$

and that A, B are compact. We claim that $\delta(A,B) = 0$. If not, then $\delta(A,B) > 0$. By the compactness of A, B there exist $a \in A$, $b \in B$ such that $\delta(A,B) = d(a,b)$. Since $f^p g^q A = A$ and $h^m t^n B = B$, we can find $x \in A$ and $y \in B$ with $f^p g^q x = a$ and $h^m t^n y = b$. It follows from (1.12) and Lemma 1.1 that

$$\begin{aligned}\delta(A,B) &= d(f^p g^q x, h^m t^n y)\\ &< \delta(\bigcup_{u\in C_{fg}} uO_{fg}(x), \bigcup_{v\in C_{ht}} vO_{ht}(y))\\ &\le \delta(\bigcup_{u\in C_{fg}} uA, \bigcup_{v\in C_{ht}} vB)\\ &= \delta(A,B)\end{aligned}$$

which is absurd and hence $\delta(A,B)=0$, which implies that $A=B=\{w\}$ for some $w\in X$. It is easy to see that

$$w = fw = gw = hw = tw = uw = vw$$

for all $u\in C_{fg}$ and $v\in C_{ht}$.

If z is another common fixed point of f,g,h and t, then $z=f^i g^i z = h^i t^i z$ for all $i\in N$. That is, $z\in A=B=\{w\}$, which proves the uniqueness of w. This completes the proof.

The following examples show that the closedness assumptions in Theorems 2.1, 2.2, 2.4 and 2.5 and Corollary 2.1 are necessary for $p+q\ge 2$ and $m+n\ge 2$.

Example 2.1 Let $X=[0,1]$ with the usual metric d. Define mappings $f,g:X\to X$ by $f0=1$, $fx=\frac{1}{3}x$ for $x\in(0,1]$ and $gx=x$ for $x\in X$. Take $q=n=0$ and $\varphi(t)=\frac{1}{3}t$ for $t\in[0,+\infty)$. It is easy to verify that

$$d(f^p x, f^m y) = \frac{1}{3}d(f^{p-1}x, f^{m-1}y) \le \varphi(\delta(O_f(x,y)))$$

for all $x,y\in X$, where $p,m\ge 2$. Thus (1.9) and (2.4) are satisfied. Note that $\frac{1}{n}\to 0$ and $f\frac{1}{n}=\frac{1}{3n}\to 0$ as $n\to\infty$ and that $0\ne 1=f0$. Hence f and fg are not closed. Thus the conditions of Theorems 2.1 and 2.2 and Corollary 2.1 are satisfied except the closedness assumption; f however has no fixed point.

Example 2.2 Let $(X,d),f,g,p,q,m,n$ be as in Example 2.1. Take $h=f$ and $t=g$. Then

$$d(f^p x, f^m y) = \frac{1}{3}d(f^{p-1}x, f^{m-1}y) < \delta(O_f(x,y))$$

for all $x,y\in X$ with $f^p x\ne f^m y$, where $p,m\ge 2$. Thus the conditions of Theorems 2.4 and 2.5 are satisfied except the closedness assumption; f however has no fixed point.

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