Delayed complex systems: An overview

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There does not exist a generally accepted definition for the notion of complex systems in science, but it is a common belief that complex systems show features which cannot be explained by just looking at their constituents. Thus, a complex system normally involves interaction of subunits, and depending on the time scales the propagation speed of information may become relevant for the dynamics. Therefore, it has been nowadays recognised that time delay may play a vital role to understand complex behaviour.

The new era of complexity science faces two challenges, namely dealing with the nontrivial topology of interacting subunits, and the challenge caused by time delay with the associated dynamics taking place in infinite dimensional phase spaces. In the past the subject of time delay dynamics has been considered as a specialists topic so that only relatively few researchers were attracted. Within the last 15 years the research activities have considerably increased as many new applications have emerged in different areas, such as electronic engineering, controlling chaos, laser physics, or neuroscience. Dynamics with time delay is going to play a vital role in new emerging fields of science and technology, for instance, because the speed of modern data processing does not allow to neglect finite propagation times of signals any more. In addition, there are striking analogies between lasers and neural systems in both of which delay effects are abundant.

It is an intriguing feature of time delay that the phase space of any ordinary differential equation subjected to delay becomes infinite dimensional. Hence, even simple equations of motion are able to produce extremely complex behaviour and bifurcation scenarios. Time delay has two complementary, counterintuitive, and almost contradicting facets. On the one hand, delay is able to induce instabilities, bifurcations of periodic and more complicated orbits, multistability, and chaotic motion. On the other hand, delay can suppress instabilities, stabilise unstable stationary or periodic states, and may control complex chaotic dynamics.

The recently held workshop on *Delayed Complex Systems* – from 5th-9th October at the Max Planck Institute for the Physics of Complex Systems, Dresden/Germany – provided a forum for such topics. We took this opportunity to assemble a list of world leading experts which now enables us to present an overview of the state of art in this field. The theme issue covers both, applications and experiments as well as mathematical foundations. The individual contributions summarise recent research results, but also address the broader context. Thus, the presentation is kept accessible for a large audience. The twelve articles cover all aspects of delay dynamics ranging from control and stability, synchronisation, and numerical approximations, with applications in laser systems, neural science, and engineering.

The first contribution by Kestutis Pyragas and coworkers addresses the issue how to use time delay for the purpose of forecasting and to optimise the prediction horizon using dynamical systems theory. The following five contributions are devoted to various aspects of synchronisation and intermittency in time delayed dynamical systems. The work by Bernold Fiedler et al. studies the synchronisation of coupled oscillators by time delayed feedback in a rigorous way. Thomas Murphy and collaborators give an overview of experimental strategies on synchronisation of coupled optical elements. The contribution of Jordi Tiana Alsina et al. extends such an analysis to quantify the degree of synchronisation in coupled semiconductor lasers using concepts of symbolic dynamics. Synchronisation of units with time delay can be efficiently used to establish a secure public key communication protocol, and the contribution by Wolfgang Kinzel and coworkers explains the fundamental ideas of such a concept. Finally the article by Sebastian Brandstetter et al. provides a comprehensive overview of coupled FitzHugh-Nagumo systems with a special emphasis on the effect of coloured noise and time delayed feedback.

The second half of the theme issue extends the scope to problems in digital engineering, the impact of imperfections and stochastic forces, as well as to biological systems. The contribution by Jason Boulet et al. addresses the problem of postural dynamics and identifies a critical time scale for the delay. The following two contributions study neural network models. The work by Gabor Orosz and coworkers describes how to use time delayed feedback to regulate the behaviour of biological networks, and the article by Jeremie Lefebvre et al. presents a bifurcation analysis for a neural network model of the sensory system. The work by Tamas Insperger and collaborators extends the focus of this issue to fundamental aspects of control in engineering, in particular, the influence of digital delay on the control performance. The penultimate contribution by Thomas Erneux and coworkers applies asymptotic methods to investigate front propagation in reaction-diffusion systems subjected to delay, and thus nicely supplements the previous studies on the FitzHugh-Nagumo model. We conclude this issue with a study on the emergence of chaos in digital circuits. Hugo de S. Calvancante et al. discuss the mechanisms for generating chaotic motion in Boolean networks.

We think this interdisciplinary theme issue may stimulate future developments and will encourage new interactions between different lines of research within this rapidly expanding vibrant field of delayed complex systems.